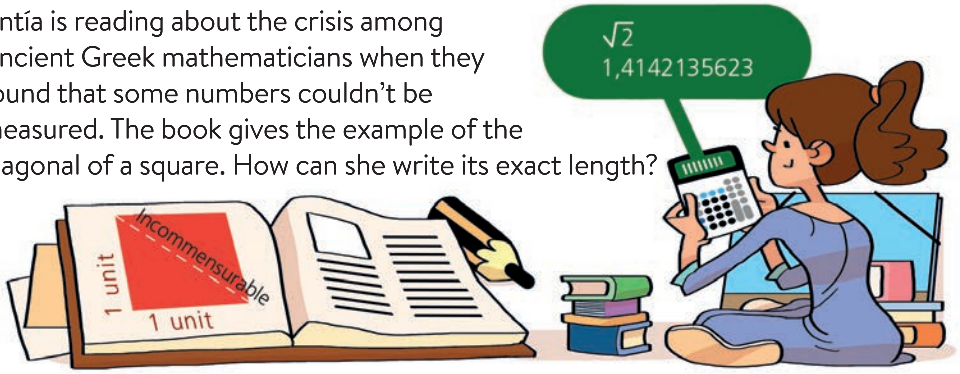
1 Rational and irrational numbers

Antía is reading about the crisis among Ancient Greek mathematicians when they found that some numbers couldn't be measured. The book gives the example of the diagonal of a square. How can she write its exact length?



The decimal expression of $\sqrt{2}$ doesn't fit in Antía's calculator display. $\sqrt{2}$ has an infinite number of decimals, and they aren't recurring.

Antía knows that rational numbers can be written as fractions of two integers. For example:

$$1 = \frac{1}{1} = \frac{-2}{-2} = \frac{73}{73} = \ldots;\ 1{,}4 = \frac{14}{10} = \frac{7}{5} = \frac{-21}{-15} = \ldots;\ 1{,}\overset{\frown}{4} = \frac{13}{9} = \frac{26}{18} = \frac{-39}{-27} = \ldots$$

A **rational number** is any number that can be expressed as a fraction, $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Antía looks at her calculator display again.

$$\sqrt{2} = 1{,}4142137309\ldots$$

Its decimal expansion is neither exact nor recurring. It can't be expressed as a fraction so it isn't a rational number. It's an irrational number.

An **irrational number** is any number that cannot be expressed as a fraction. Its decimal expression is non-terminating and non-repeating.

Real numbers

All the numbers we know and use are classified in groups that have been expanded and organised over time. They are all real numbers.

All the rational and irrational numbers together make up a number set called **Real numbers**. The symbol $\mathbb{R}$ is used for the set of real numbers.

Take note

Exact → Rational

$\sqrt{2} \neq 1{,}4142135623$

Irrational

$\sqrt{2} = 1{,}4142125623\ldots$

Take note

The set of real numbers $\mathbb{R}$ contains the set of rational numbers $\mathbb{Q}$.

This in turn contains the set of integers $\mathbb{Z}$.

And the integers contain the natural numbers $\mathbb{N}$.

Worked example

1 **Classify these numbers as rational or irrational:** $-34{,}5678910\ldots \quad 34{,}5678 \quad \frac{3}{2} \quad \sqrt{7} \quad \sqrt{9} \quad -19$

Solution

- Rational: $34{,}5678$ (exact decimal), $\frac{3}{2}$ (fraction), $\sqrt{9} = 3$ (natural), -19 (integer) because they can be expressed as fractions.
- Irrational: $-34{,}5678910\ldots$, $\sqrt{7}$ because their decimal expansions are non-terminating and non-repeating.

Activities

2 **Write the decimal expansion of these fractions in your notebook. What type of decimal number is each one?**

a) $\frac{-35}{8}$ c) $\frac{23}{7}$

b) $\frac{31}{11}$ d) $-\frac{17}{6}$

Look at your working and explain why a fraction cannot be an irrational number.

3 **Copy these numbers in your notebook and classify them according to the type of decimal number they are. Find the generating fraction for each one.**

a) $3{,}24$ b) $3{,}2\widehat{4}$ c) $3{,}\widehat{24}$ d) $32{,}\widehat{4}$

4 **Which of these decimal numbers can be expressed as fractions? Find the fraction when possible.**

a) 5,010101... d) −5,010110111...

b) −6,1811881... e) 7,8999999...

c) 93,5353535... f) −0,1357911...

5 **Classify these numbers as rational or irrational.**

3,7 $\sqrt{18}$ $\sqrt[3]{5}$ −4,3525252... $\sqrt{\frac{8}{2}}$

Take note

The Natural numbers $\mathbb{N}$ are the counting numbers: 1, 2, 3, ...

The Integers $\mathbb{Z}$ are all the positive and negative whole numbers, and zero.

The rational numbers $\mathbb{Q}$ are all the numbers that can be expressed as fractions.

The Real numbers $\mathbb{R}$ include all rational and irrational numbers.

6 **In your notebook, draw the number sets diagram and put these numbers in the correct sets.**

$\sqrt{\frac{9}{4}}$ $-\frac{56}{7}$ 13 $\sqrt{3}$ $\sqrt{121}$

7 **What digit is in the tenth decimal place in each of these numbers?**

a) $\frac{53}{11}$ b) $\frac{\pi}{11}$

What differences do you notice? What causes them?

8 **Study the numbers below, then answer this question:** ***What type of number can be a square root of a fraction?***

$\sqrt{\frac{5}{4}}$ $\sqrt{\frac{64}{25}}$ $\sqrt{\frac{36}{9}}$ $\sqrt{\frac{18}{2}}$

9 **Calculate the results of these operations using the decimal expansion first, then using fractions. Do you get the same result in each case?**

a) 2,8666... + 3,2222...

b) 1,8333... + (−1,6)

c) −4,3333... · 3

d) 5,6262... · 0,5

Think about this question: *Are the sum and product of rational numbers also rational numbers?*

10 **True or false?** ***The sum of two irrational numbers can be a rational number.***

Justify your answer with examples.

11 **In each case, is the length of the diagonal of the rectangle a rational or irrational number?**

a) The rectangle's base is $\frac{3}{4}$ cm and its height is 1 cm.

b) The rectangle's base is $\frac{1}{2}$ cm and its height is 1 cm.

12 **This table shows the results of an election for a class representative. There are fewer than 35 students in the class, and each student has exactly one vote.**

	Sofía	Raúl	Blank
Vote share	$63{,}\widehat{63}$ %	$33{,}\widehat{3}$ %	The rest

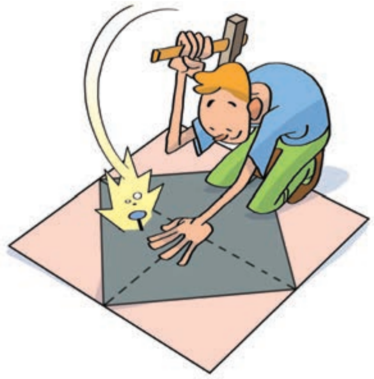
a) What percentage of votes were blank?

b) How many students are in the class? How many students voted for Sofía? And for Raúl?

CLIL zone

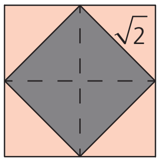
13 **Listen to the students talking about the following statement, and then decide which student is correct.**

The volume of a cuboid that measures $\sqrt{5}$ by $\sqrt{8}$ by $\sqrt{10}$ will be an irrational number.



2 Representing numbers on a number line

Ernesto made a mural with four square planks, each measuring 1 m × 1 m. He wants to place a metal square on the diagonals of the four squares. He knows the metal square needs to measure exactly $\sqrt{2}$ m × $\sqrt{2}$ m.



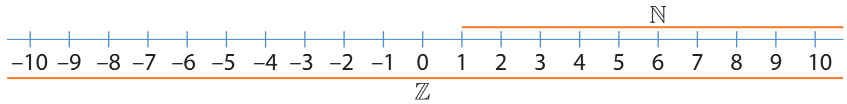
The shop sold him a 1,5 m square and Ernesto is sure it won't fit. He places it on top of the mural to check. He's correct: the metal square is too big.

$1{,}5 - \sqrt{2} > 0 \rightarrow 1{,}5$ is bigger than $\sqrt{2}$

> Given any two real numbers, a and b: a is greater than b if $a - b > 0$, a is less than b if $a - b < 0$ and a is equal to b if $a - b = 0$.

When Ernesto puts the metal on the mural so that the vertices are aligned, part of the metal sticks out. How can he represent this?

This is how we represent integers on a number line:



To represent rational numbers exactly, we can use Thales' theorem and draw parallel lines to divide one unit into equal parts.

If the number is irrational but can be expressed as a square root, we can use Pythagoras' theorem to represent it exactly on the number line.

Ernesto can now calculate the exact lengths of the diagonals and sides of the metal square. This will tell him how much he needs to cut off to make his mural.

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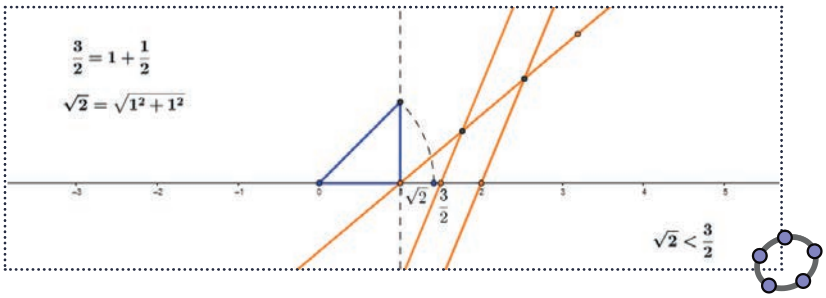


> To construct a **real number line**, draw a horizontal line, choose a point to represent 0 (the origin) and a unit length. Each real number is associated with a unique point on the line, and each point is associated with a unique real number. Two numbers on the same point are equal.

Take note

To represent a fraction on a number line, first simplify it, if necessary, so that it is in its lowest terms.

Lost in translation

It makes sense to use ℕ for the set of Natural numbers and ℝ for the set of Real numbers, but why do we use ℚ for rational numbers and ℤ for integers? Both come from German words: *Quotient* means ratio and *Zahlen* means numbers.

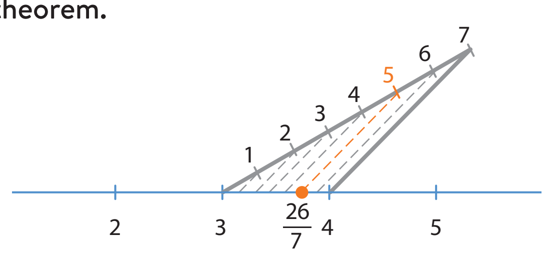
Worked example

14 **Represent the rational number $\frac{26}{7}$ on a number line using Thales' theorem.**

Solution

$\frac{26}{7} = 3 + \frac{5}{7} \rightarrow 3 < \frac{26}{7} < 4$

$\frac{26}{7}$ lies between 3 and 4 on the real number line.



Activities

15 Copy these pairs of numbers in your notebook and say which number is greater in each pair.

a) $2{,}3101$ and $2{,}3\overset{\frown}{10}$

b) $12{,}92\overset{\frown}{3}$ and $12{,}9\overset{\frown}{23}$

c) $-5{,}828228222...$ and $-5{,}\overset{\frown}{82}$

d) $-7{,}1\overset{\frown}{45}$ and $-7{,}14\overset{\frown}{54}$

e) $-\frac{39}{11}$ and $-\frac{25}{7}$

f) $\sqrt{44}$ and $\frac{73}{11}$

16 Put these numbers in order from smallest to largest.

$3{,}2\overset{\frown}{32}$ $\quad -3{,}2\overset{\frown}{23}$ $\quad -3{,}223344...$

$3{,}233$ $\quad -3{,}2\overset{\frown}{3}$ $\quad 3{,}23$ $\quad 3{,}232233...$

17 Put these numbers in order from largest to smallest.

$\sqrt[3]{19}$ $\quad -\frac{34}{13}$ $\quad \frac{19}{7}$

$2{,}64\overset{\frown}{57}$ $\quad -\sqrt{7}$ $\quad -2{,}6\overset{\frown}{45}$

18 Represent these numbers on the real number line.

a) $\frac{3}{7}$ b) $\frac{17}{7}$ c) $-\frac{5}{7}$ d) $-\frac{12}{7}$

19 Draw a number line in your notebook and represent these numbers on it. Then write them in order from smallest to largest.

a) $\frac{6}{5}, -\frac{3}{8}, -\frac{26}{3}, \frac{17}{6}$

b) $-\frac{30}{8}, -\frac{26}{7}, -\frac{19}{5}, -\frac{22}{6}$

20 Represent these irrational numbers exactly on the real number line.

a) $\sqrt{10}$ b) $\sqrt{17}$ c) $\sqrt{8}$

21 Represent $\sqrt{18}$ exactly on the real number line. Explain the changes you must make to represent $-\sqrt{18}$ and add it to the line too.

22 Represent these numbers exactly on the real number line, choosing the most appropriate method in each case.

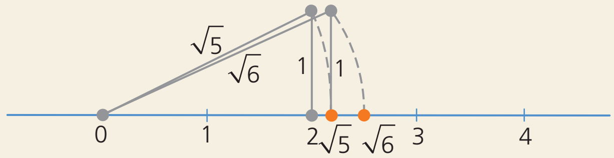
a) $-3{,}25$ b) $4{,}16$ c) $-\sqrt{13}$

23 Represent the number $\pi = 3{,}14159265358979...$ on the real number line. How can you do this? Is it possible to represent it exactly?

24 How could you represent the irrational number $2\sqrt{10}$ exactly on the real number line? Write the steps required in order in your notebook.

25 Represent $\sqrt{17}$ on the real number line. Use Thales' theorem to represent $\frac{\sqrt{17}}{3}$. Explain how you do it in your notebook.

26 Look at this representation of $\sqrt{6}$. In your notebook, explain how it was done, step by step.



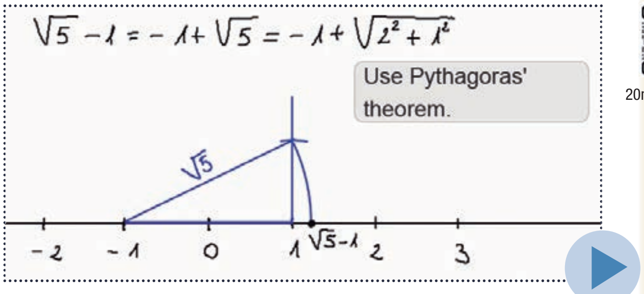
27 Look at the previous activity and represent these numbers on the real number line:

a) $\sqrt{7}$ b) $\sqrt{11}$ c) $\sqrt{14}$

Worked example

28 Represent the number $\sqrt{5} - 1$ on the real number line.

Solution



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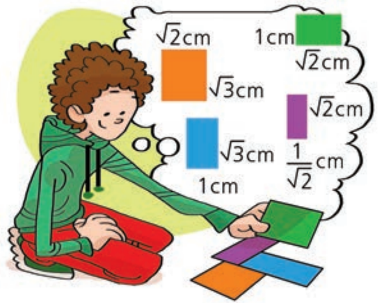
29 Represent these irrational numbers on the real number line.

a) $2 + \sqrt{5}$ b) $-3 + \sqrt{2}$ c) $\frac{3 + \sqrt{10}}{2}$

CLIL zone

30 Listen to the descriptions of the coordinate points used to represent the following numbers on the real number line and match to the correct number.

1) $\sqrt{29}$ 2) $\sqrt{17}$ 3) $\sqrt{50}$ 4) $\sqrt{41}$ 5) $\sqrt{5} - 2$ 6) $\sqrt{13} + 2$ 7) $3\sqrt{2} + 1$ 8) $\sqrt{61} - 3$



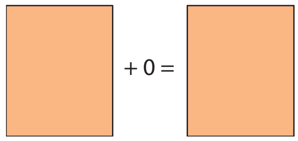
3 Properties of addition and multiplication

Diego is making a mosaic. He has discovered various properties.

Properties of addition

Neutral element: 0

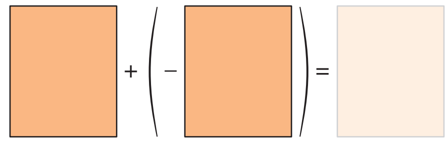
If Diego doesn't add any pieces, the length does not change.



$$\sqrt{2} + 0 = \sqrt{2}$$

Opposite element: −*a*

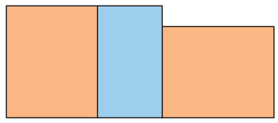
Removing a piece after adding it doesn't affect the result.



$$\sqrt{2} + \left(-\sqrt{2}\right) = 0$$

Associative: $a + (b + c) = (a + b) + c$

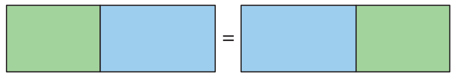
The sum of the lengths is the same, regardless of how Diego groups the pieces.



$$\sqrt{2} + \left(1 + \sqrt{3}\right) = \left(\sqrt{2} + 1\right) + \sqrt{3}$$

Commutative: $a + b = b + a$

Changing the order of the pieces doesn't affect the length.



$$\sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$$

Properties of multiplication

Unit element: 1

The area of the piece with a height of 1 cm is the same as the base.

$$A = \sqrt{2} \cdot 1 = \sqrt{2} \text{ cm}^2$$

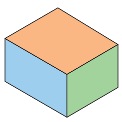
Reciprocal element: $a^{-1} = \frac{1}{a}$

The area of the piece measuring $\sqrt{2}$ cm $\cdot \frac{1}{\sqrt{2}}$ cm is 1 cm^2.

$$A = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1 \text{ cm}^2$$

Associative: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

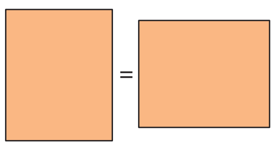
The volume doesn't depend on the order in which you multiply the three dimensions.



$$\sqrt{3} \cdot \left(\sqrt{2} \cdot 1\right) = \left(\sqrt{3} \cdot \sqrt{2}\right) \cdot 1$$

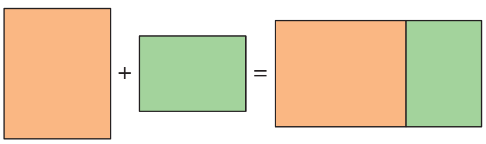
Commutative: $a \cdot b = b \cdot a$

The area is the same regardless of the orientation of the piece.



$$\sqrt{2} \cdot \sqrt{3} = \sqrt{3} \cdot \sqrt{2}$$

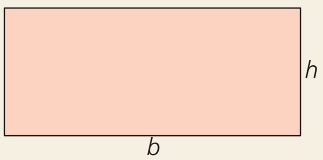
If Diego wants to calculate the area of two pieces at the same time, he can do it in two ways, using the **distributive property:** $a \cdot (b + c) = a \cdot b + a \cdot c$



$$\sqrt{2} \cdot \left(\sqrt{3} + 1\right) = \sqrt{2} \cdot \sqrt{3} + \sqrt{2} \cdot 1$$

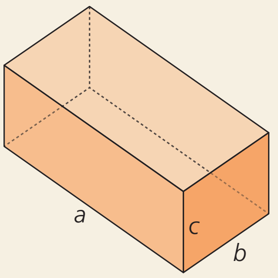
Remember

Area of a rectangle



$$A = b \cdot h$$

Volume of a cuboid



$$V = a \cdot b \cdot c$$

Take note

Zero doesn't have a reciprocal because no number multiplied by zero can equal 1.

Activities

31 Write the opposites of these numbers in your notebook.

a) $2+\sqrt{3}$ c) $\dfrac{\pi}{6}$ e) $-\dfrac{7}{18}$

b) 0 d) $0{,}1\overset{\frown}{6}$ f) $1-\sqrt{5}$

32 Write the opposites of $\sqrt{7}$ and $-\dfrac{31}{7}$ in your notebook and represent them exactly on the real number line. Now answer these questions:

a) Which of the two numbers is smaller in each case?

b) What distance from zero separates a number and its opposite? What is the total distance between them?

33 Does a real number always have an opposite? And a rational number? Is there a number set in which a number doesn't have an opposite? If so, which one?

34 Calculate the reciprocal of these numbers.

$-4{,}67 \qquad \sqrt{5} \qquad \dfrac{1}{3} \qquad -7 \qquad \dfrac{\pi}{4}$

35 Consider these numbers.

$-1{,}75 \qquad 0{,}0\overset{\frown}{45} \qquad -5 \qquad 0{,}\overset{\frown}{3}$

- Express them as fractions and calculate their reciprocals. Does the sign change?
- Calculate the decimal expansions of the reciprocals.
- Represent each number and its reciprocal on a number line and compare them with 1 and −1. What do you notice?

36 Consider the different number sets and find the reciprocal of one of their elements (not zero).

a) In which cases does the reciprocal belong to the same set? When doesn't it belong to the same set?

b) Is there any element that has a reciprocal in all the number sets? What is it? What is it called?

c) In which sets is there a reciprocal for all numbers?

37 In your notebook, write the steps for finding the reciprocal of a fraction. Does this correspond with the reciprocal element property?

Take note

We can use the distributive property to factor out the common factor in an expression.

Distributive property →

$a\cdot(b+c)=a\cdot b+a\cdot c$

← **Common factor**

Worked example

38 Factor out the common factor and then calculate.

a) $2\cdot\dfrac{3}{5}+2\cdot\left(-\dfrac{3}{4}\right)+2-\dfrac{14}{5}$

b) $-\dfrac{3}{4}+\dfrac{21}{2}+\dfrac{9}{4}-\dfrac{15}{2}$

Solution

a) $2\cdot\dfrac{3}{5}+2\cdot\left(-\dfrac{3}{4}\right)+2\cdot 1-2\cdot\dfrac{7}{5}$

$=2\cdot\left(\dfrac{3}{5}-\dfrac{3}{4}+1-\dfrac{7}{5}\right)$

$=2\cdot\dfrac{(12-15+20-28)}{20}=2\cdot\dfrac{-11}{20}=-\dfrac{11}{10}$

b) $-\dfrac{3\cdot 1}{2\cdot 2}+\dfrac{3\cdot 7}{2\cdot 1}+\dfrac{3\cdot 3}{2\cdot 2}-\dfrac{3\cdot 5}{2\cdot 1}=$

$=\dfrac{3}{2}\cdot\left(-\dfrac{1}{2}+7+\dfrac{3}{2}-5\right)=\dfrac{3}{2}\cdot 3=\dfrac{9}{2}$

39 Calculate, following the correct order of operations and factoring out the common factor. Discuss the advantage of factoring in each case.

a) $55+77-22+11-33$

b) $\dfrac{2}{15}-\dfrac{8}{25}+\dfrac{14}{30}$

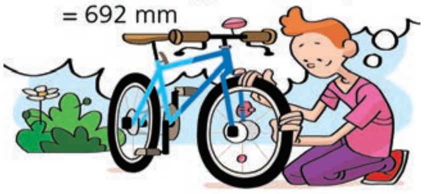
c) $6\pi-10\pi+45\pi-15\pi$

CLIL zone

40 Listen and write the expression. Then, follow the instructions to simplify.

4 Approximations and errors

622 mm (inside diameter) +
+ 2·35 mm (tyre thickness) =
= 692 mm



Mathematical language

To indicate that a number is approximate, we use the ≈ symbol.

$\pi \approx 3{,}1416$

Take note

When we round a large number, such as 2 873, we must pay attention to the number of significant figures required:

- to 1 significant figure: 3 000
- to 2 significant figures: 2 900
- to 3 significant figures: 2 870

Take note

The absolute error bound will have one more significant figure than the approximation, with a 5 in the last place and zeros in all the other places.

Take note

The more significant figures, the lower the relative error.

Alberto is calibrating the odometer on his bike so that he can measure how far he cycles as accurately as possible. He calculates the circumference of the wheel: $C = \pi \cdot d = \pi \cdot 0{,}692 \text{ m} = 2{,}1739821... \text{ m}$

He doesn't need that many decimal places, so he uses an **approximation**.

	Tenths	Hundredths	Thousandths	Ten thousandths
Lower bound	2,1	2,17	2,173	2,1739
Upper bound	2,2	2,18	2,174	2,1740
Rounded	2,2	2,17	2,174	2,1740

Alberto rounds to three decimal places: $C \approx 2{,}174$ m

This number has four significant figures. Three are exact and one is approximate.

> The **significant figures** in an approximation are the ones that carry meaning and contribute to the number's precision.

The difference between the exact value and the odometer reading is the absolute error.

- Alberto calculates the **absolute error** in metres.

  Absolute error, $E_a = |0{,}692\pi - 2{,}174| = 0{,}0000178...$ m

  He can't calculate the exact error because π is an irrational number, so he estimates the **error bound**: Absolute error bound < 0,0005 m

- Alberto doesn't know the true value, so he also calculates the **relative error bound** by dividing the absolute error bound by the approximation.

  $$\text{Relative error bound} = \frac{E_a}{|x|} < \frac{0{,}0005}{2{,}174} = 0{,}00022999... < 0{,}0003$$

  The **relative error** is less than 0,03%

> - The **absolute error** is the difference, expressed as an absolute value, between a true value, x, and its approximation, a: $E_a = |x - a|$
>   It has the same units as the true value. If the true value is unknown, an **absolute error bound** can be estimated.
> - The **relative error** is the quotient between the absolute error and the true value.
> $$E_r = \frac{E_a}{|x|}$$
>   To find the **relative error bound**, calculate the quotient between the absolute error bound and the approximation. It's usually expressed as a percentage.

Worked example

41 **Calculate the error bounds when the volume of a sphere with a radius of 1 m is approximated in dm³.**

Solution

$V_{\text{sphere}} = \frac{4}{3}\pi r^3 \approx 4{,}189 \text{ m}^3$ $\quad E_a = |x - a| < 0{,}0005 \text{ m}^3$ $\quad E_r = \frac{E_a}{|x|} < \frac{0{,}0005}{4{,}189} = 0{,}0001193... < 0{,}0002 = 0{,}02\ \%$

Activities

42 Calculate the absolute and relative errors in these approximations.

a) $\frac{2}{3} \approx 0{,}67$ **b)** $\sqrt{3} \approx 1{,}73$ **c)** $\pi \approx 3{,}14$

43 Find the upper and lower approximations of $\frac{15}{13}$ and $\sqrt{27}$ to 1, 2 and 3 decimal places.

a) Calculate the absolute error in each case.

b) Use the results to explain the usefulness of rounded approximations.

44 How would you express these quantities in an informal conversation? Write how many significant figures you would use and calculate the absolute and relative error bounds.

a) Cost of resurfacing a road: €134 988,33

b) Price of a car: €16 685,37

c) Size of a virus: 0,008375 mm

45 Look at the profit made by a company in these two periods and approximate each amount to three significant figures.

Period	Profit (€)
Month	7 238,34
Year	95 357,81

Now calculate the absolute and relative error bounds in each case, in euros.

Repeat the calculations for an approximation to two significant figures, analyse the results and answer these questions:

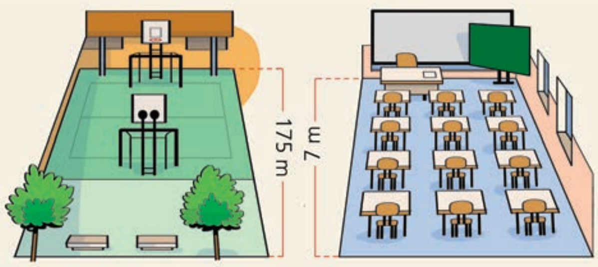
a) What are the errors in each case?

b) Is there a link between the number of significant figures and the relative error?

46 Calculate the absolute and relative error bounds of these approximated distances.

Journey	Distance (km)
Huelva – Sevilla	90
A Coruña – Logroño	500
Cádiz – Girona	1 200

47 Compare these two estimations.

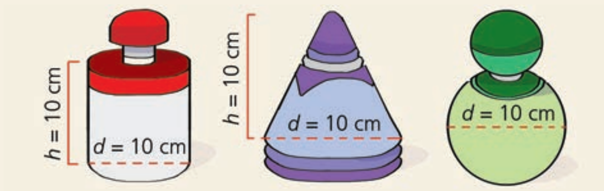


a) How many significant figures does each one have?

b) Estimate the absolute error bound for each one. What do you notice?

c) Calculate the relative error bound for each one. Are the two approximations equally reasonable?

48 Calculate the area and volume of each bottle, giving the true and approximate values in each case. How many significant figures have you used? Calculate the error bounds. Remember: 1 L = 1 dm^3



Based on your results, decide which bottle is best. Think about the surface area, volume, storage, shape, aesthetics, and so on.

CLIL zone

49 Work with a partner. In each scenario, use the phrases below to determine who has made the greater absolute error and the greater relative error.

a) Roberto says the volume of a box measuring 3,2 cm by 7,5 cm by 16,4 cm is 390 cm^3.
Luis says the volume of a box measuring 8,7 cm by 13,1 cm by 11,6 cm is 1320 cm^3.

b) Candide says the surface area of a sphere with a radius of 5 cm is 320 cm^2.
Erica says the surface area of a sphere with a radius of 16 cm is 3200 cm^2.

First, we need to calculate the true... Then, the amount that the student found...
Roberto/Luis/Candide/Erica's absolute error is... His/her relative error is...

5 Intervals

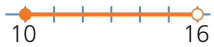
Lourdes is looking at the cost of using water in her home.

Tariff	Monthly usage	Rate
Tariff 1	Less than 10 m^3	€0,2965/m^3
Tariff 2	10 m^3 to less than 16 m^3	€0,5486/m^3
Tariff 3	16 m^3 to less than 18 m^3	€0,6855/m^3
Tariff 4	18 m^3 or more	€1,3163/m^3

The average water consumption is 100 L per person per day, i.e. less than 10 m^3 per month.

Lourdes will pay the minimum rate if she uses less than 10 m^3. If she uses more than that, she will move to a higher tariff. Tariff 2 is for water consumption, x, of 10 m^3 to less than 16 m^3. This is an **interval** and is written like this:

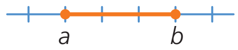
$[10, 16) = \{x \in \mathbb{R} \ / \ 10 \leq x < 16\}$



An **interval** is a set of real numbers between two values called **endpoints**. A filled circle indicates the endpoint value is included. An empty circle indicates the endpoint value is not included.

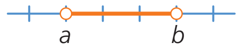
Intervals can be classified in four types according to whether or not they include their endpoints.

Closed interval



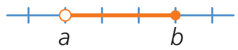
$[a, b] = \{x \in \mathbb{R} \ / \ a \leq x \leq b\}$

Open interval



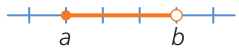
$(a, b) = \{x \in \mathbb{R} \ / \ a < x < b\}$

Left-open



$(a, b] = \{x \in \mathbb{R} \ / \ a < x \leq b\}$

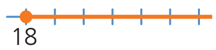
Right-open



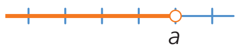
$[a, b) = \{x \in \mathbb{R} \ / \ a \leq x < b\}$

Tariff 4 only gives one number, 18 m^3 or more. Any usage equal to or greater than this amount will incur the highest rate. This tariff has only one endpoint, the minimum value.

$[18, +\infty) = \{x \in \mathbb{R} \ / \ x \geq 18\}$



These are intervals too.



Real numbers less than a.

$(-\infty, a) = \{x \in \mathbb{R} \ / \ x < a\}$

Real numbers greater than a.

$(a, +\infty) = \{x \in \mathbb{R} \ / \ x > a\}$

Real numbers less than or equal to a.

$(-\infty, a] = \{x \in \mathbb{R} \ / \ x \leq a\}$

Real numbers greater than or equal to a.

$[a, +\infty) = \{x \in \mathbb{R} \ / \ x \geq a\}$

Mathematical language

To indicate that a number is in an interval, we use the $\in$ symbol:

$3 \in (1, 7)$

We say this as: *3 belongs to the interval (1, 7).*

Take note

A value is included if it's preceded or followed by a square bracket.

A value is excluded if it is preceded or followed by a round bracket.

[3, 7) means the interval from 3 to 7, where 3 is included but 7 is not.

Mathematical language

The real number line can also be expressed as an interval:

$\mathbb{R} = (-\infty, +\infty)$

Similarly:

$\mathbb{R}^- = (-\infty, 0)$

$\mathbb{R}^+ = (0, +\infty)$

Activities

50 Express these sets as intervals in your notebook. What type of interval are they?

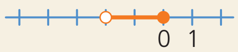
a) Numbers between -3 and 2, including 2.

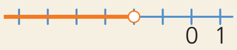
b) Numbers less than or equal to -2.

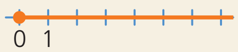
c) Positive numbers less than 5.

d) Numbers greater than -3.

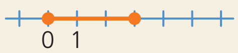
51 Express these number sets as intervals. Use x for the real numbers that belong to each set.

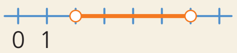
a) 

d) 

b) 

e) 

c) 

f) 

52 Show these sets on number lines, noting their endpoints. Write the inequality next to each number line.

a) $[2, +\infty)$ b) $(-5, -1]$ c) $(-\infty, 3)$ d) $[0, 7]$

53 Express these inequalities as intervals and show them on number lines.

a) $\{x \in \mathbb{R} \,/\, -5 < x < 1\}$ c) $\{x \in \mathbb{R} \,/\, x \leq -1\}$

b) $\{x \in \mathbb{R} \,/\, x > -5\}$ d) $\{x \in \mathbb{R} \,/\, -1 \leq x < 5\}$

54 Show these inequalities on number lines and express them as intervals. What do you notice?

a) $\{x \in \mathbb{R} \,/\, x \geq -3\}$ b) $\{x \in \mathbb{R} \,/\, -3 \leq x\}$

55 Express these inequalities as intervals:

a) $2x < 0$ c) $2 - x \geq 0$

b) $x - 3 > 0$ d) $0 < x + 1 \leq 2$

56 Determine for which numbers it's possible to calculate $\sqrt{x-2}$. Express them as an inequality and an interval, and draw them on a number line.

Worked example

57 On a number line, draw the interval formed by x.

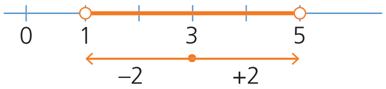
$$|x - 3| < 2$$

Solution

A number whose difference with 3 has an absolute value of 2 is two units greater or two units less than 3.

$5 - 3 = 2$ $1 - 3 = -2$

The difference will be less than 2 if the number is between 1 and 5.



$\{x \in \mathbb{R} \,/\, 1 < x < 5\} = (1, 5)$

58 Show these intervals on number lines.

a) $|x - 4| < 3$ c) $|x - 6| \leq 1$

b) $|x + 1| < 5$ d) $|x + 3| \leq 7$

59 Write open intervals that include $\sqrt{27}$ between:

a) two integers.

b) two numbers with one decimal place.

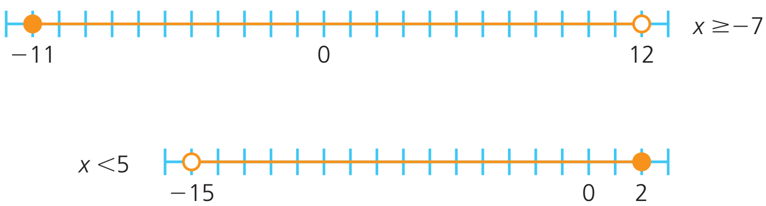
c) two numbers with two decimal places.

- What numbers do you use as the interval endpoints?
- Do you think you could find even smaller intervals?

60 Write intervals like the ones in the previous activity for the number 2,1. Is it possible?

CLIL zone

61 Sophie is trying to guess a number that Pedro has chosen. He says his number can be found using the following information. Use the phrases below to help Sophie work out if it's possible to determine Pedro's number.



Sophie must start by drawing a number line from... to...
Then she must eliminate intervals for each...
It is/isn't possible to know...