1 Natural numbers: divisibility

A restaurant uses 30 eggs to prepare 5 or 6 omelettes. The chef can make 6 omelettes containing 5 eggs each, or 5 omelettes containing 6 eggs each.

$$\begin{array}{r|l} 30 & 6 \\ \hline 0 & 5 \end{array} \qquad \begin{array}{r|l} 30 & 5 \\ \hline 0 & 6 \end{array}$$

The remainder of these divisions is 0, so 30 is divisible by 6 and by 5.
30 is a **multiple** of 6 and of 5. 6 and 5 are **divisors** of 30.

- A number, a, is **divisible** by a number, b, if the division $a : b$ is exact (it has no remainder).
- The number a is a **multiple** of b, and the number b is a **divisor** of a.

Remember

A **number is prime** if it has only two divisors: 1 and itself.

If a number isn't prime, it's **composite.**

The first prime numbers are: 2, 3, 5, 7, 11, 13 and 17.

Lost in translation

In English we use the symbol ÷ for division. So we write a divided by b as $a \div b$.

Mathematical language

Prime factor decomposition means breaking down a number into its prime factors.

Prime factor decomposition

We can write 30 as a multiplication in several ways.

Prime or composite factors	Prime factors
$30 = 6 \cdot 5$	$30 = 2 \cdot 3 \cdot 5$

To **break a number down into prime factors**, follow these steps.

$$\begin{array}{r|l} 140 & 2 \\ 70 & 2 \\ 35 & 5 \\ 7 & 7 \\ 1 & \end{array}$$

1. Write the number and draw a vertical line to the right of it.
2. Find the smallest prime number that is a divisor of the number. Write it to the right of the line.
3. Write the quotient underneath the original number.
4. Repeat the process until the quotient is 1.

The number's **prime factor decomposition** is the product of the prime numbers obtained: $140 = 2 \cdot 2 \cdot 5 \cdot 7 = 2^2 \cdot 5 \cdot 7$

Greatest common divisor and lowest common multiple

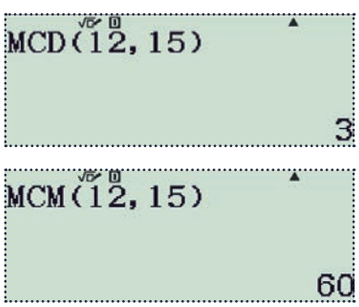
To calculate the **greatest common divisor** (g.c.d.) and **lowest common multiple** (l.c.m.) of several numbers, follow these steps.

g.c.d. (72, 108, 270)	l.c.m. (72, 108, 270)
1. Break down the numbers into prime factors. $72 = 2^3 \cdot 3^2$ $108 = 2^2 \cdot 3^3$ $270 = 2 \cdot 3^3 \cdot 5$	
2. Select the common factors raised to the smallest power. $2^3 \cdot 3^2$ $2^2 \cdot 3^3$ $2 \cdot 3^3 \cdot 5$	2. Select the common and non-common factors raised to the largest power. $2^3 \cdot 3^2$ $2^2 \cdot 3^3$ $2 \cdot 3^3 \cdot 5$
3. The product of those factors is the greatest common divisor. g.c.d. (72, 108, 270) $= 2 \cdot 3^2$	3. The product of those factors is the lowest common multiple. l.c.m. (72, 108, 270) $= 2^3 \cdot 3^3 \cdot 5$

- The **greatest common divisor** of several numbers is the largest of the common divisors.
- The **lowest common multiple** of several numbers is the smallest of the common multiples.

Using a calculator

Use the MCD and MCM buttons to calculate the greatest common divisor and lowest common multiple of two numbers.



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Activities

1 Is the larger number divisible by the smaller number? Write your answers in your notebook.

a) 14 and 168 c) 84 and 7
b) 12 and 98 d) 17 and 51

2 Write two multiples and two divisors of:

a) 24 b) 35 c) 144 d) 72

3 Break down into prime factors.

a) 66 b) 160 c) 168 d) 450 e) 392 f) 147

4 Write the prime factor decomposition of:

a) 253 b) 169 c) 187 d) 242

5 Find the greatest common divisor of:

a) 81 and 108 c) 64 and 88
b) 56 and 84 d) 168 and 216

6 Find the lowest common multiple of:

a) 27 and 36 c) 72 and 100
b) 126 and 392 d) 154 and 175

7 Find the greatest common divisor and lowest common multiple of these decompositions.

a) $2^2 \cdot 3^4 \cdot 5$ $2^3 \cdot 3 \cdot 5$ $2^3 \cdot 3^2 \cdot 7$
b) $7^4 \cdot 13^2 \cdot 23$ $2^5 \cdot 7^3 \cdot 23^2$ $3^2 \cdot 7^4 \cdot 23^3$

8 Find the greatest common divisor and lowest common multiple of these numbers.

a) 112, 168 and 196 c) 243, 240 and 294
b) 216, 441 and 225 d) 720, 468 and 504

Take note

If a set of numbers has no common factors, the greatest common divisor of those numbers is 1. The numbers are said to be **relatively prime**, or **coprime**.

Worked example

9 Are 45 and 56 relatively prime?

Solution

Break down the two numbers into prime factors.

$45 = 3^2 \cdot 5$ $56 = 2^3 \cdot 7$

They have no common factors, so the g.c.d. is 1 and the numbers are relatively prime.

10 Are these pairs of numbers relatively prime?

a) 112 and 297
b) 117 and 130
c) 245 and 324
d) 170 and 375

11 Óscar has bought several dozen eggs. How many eggs has he bought if the number is between 90 and 100?

12 A baker has 312 madeleines, and she wants to sell them in bags.

a) Can she put 7 madeleines in each bag with none left over?
b) What about 8 madeleines in each bag?
c) If the baker wants to sell equal bags of between 5 and 15 madeleines, what are her options?

13 María has programmed the lights on her shop sign so that the blue neon flashes every 28 seconds, and the green neon flashes every 42 seconds. If the two neons are both on now, how much time must pass until the two neons flash simultaneously again?

14 Carol has three strings of flashing Christmas lights. The red lights flash every 32 seconds, the green lights flash every 24 seconds, and the blue lights flash every 18 seconds. If the lights are connected to the power at the same time, how much time must pass before all three strings flash at the same time?

15 A construction company has some aluminium panels. 25 of the panels are 1,20 metres, 32 of the panels are 1,50 metres and 12 of the panels are 1,80 metres. The company decides to cut the panels into equal pieces of the longest possible length.

a) What is the longest possible length?
b) How many panels will the company have after cutting them to that length?

Lost in translation

In English we use a decimal point (.) to separate the integer part of a number from its decimal part. For example, we write 5,5 in Spanish as 5.5 in English.

CLIL zone

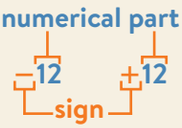
16 Listen to the information about three bells ringing in a town. Use prime factor decomposition and lowest common multiples to work out when the three bells will next ring at the same time.

2 Positive and negative numbers

Ordering integers

Remember

Integers are made up of a sign and a numerical part.



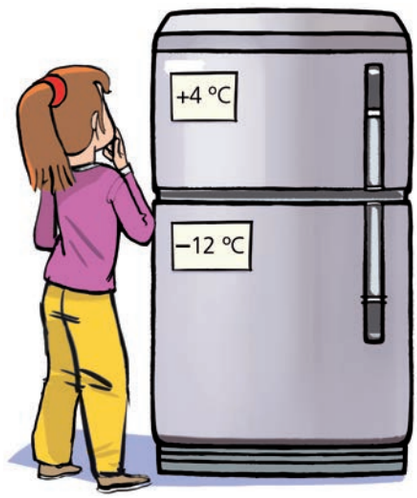
Leonor thinks that her fridge-freezer is broken. She's not sure if the fridge and freezer compartments are cold enough.

The instruction manual says that:

- the optimum temperature for the fridge compartment is between +3 °C and +5 °C.
- the optimum temperature for the freezer compartment is −18 °C.

She looks at the temperatures given by the indicators on the fridge-freezer.

- The fridge temperature indicator shows +4 °C.
- The freezer temperature indicator shows −12 °C.

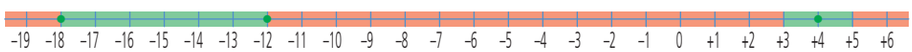


Mathematical language

- The set of natural numbers is represented by the letter $\mathbb{N}$.
- The set of integers is represented by the letter $\mathbb{Z}$.

Not working properly
$-18 < -12$

Working properly
$+3 < +4 < +5$



As $+3 < +4 < +5$, the fridge compartment is working perfectly. However, as $-12 > -18$, the freezer compartment isn't working properly. It's 6 degrees warmer than the optimum temperature.

To **compare two integers**, use these rules.

- If both are positive, the number further from 0 on a number line is larger.
- If one number is positive and the other negative, the positive number is larger.
- If both are negative, the number closer to 0 on a number line is larger.

Absolute and opposite values of integers

Mathematical language

- The symbol | | is used for absolute values.

$|+15| = 15$ $\quad |-12| = 12$

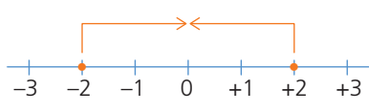
- The opposite of a number is indicated by the letters op with the number in brackets.

$\text{op}(+5) = -5$ $\quad \text{op}(-7) = +7$

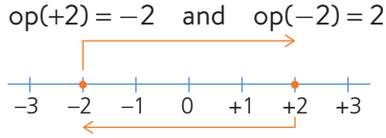
The temperature indicator on Leonor's fridge-freezer is broken now. It doesn't show if the temperature is positive or negative. This means that if the fridge compartment indicator says 2 °C, the actual temperature could be +2 °C or −2 °C.

The two numbers have the same **absolute value**, but they're **opposite** numbers.

$|-2| = 2$ and $|+2| = 2$



$\text{op}(+2) = -2$ and $\text{op}(-2) = 2$



- The **absolute value** of an integer is the distance from 0 on a number line.
- The **opposite** of an integer is another integer located at the same distance from 0 on a number line, but with the opposite sign.

Activities

17 Express each of these situations using an integer. Write your answers in your notebook.

a) Yesterday I dived to a depth of 4 metres.

b) Brrr! The thermometer says it's 3 degrees below zero this morning.

c) We've fixed the freezer and now the temperature is 18 degrees below zero.

> **Take note**
>
> Positive integers are usually written without the + sign.
>
> $+8 = 8$

18 Copy and complete using $>$ or $<$.

a) $3 \square 8$

b) $-12 \square -11$

c) $-3 \square 9$

d) $-5 \square -7$

e) $-6 \square -2$

f) $5 \square -1$

19 Order these numbers from smallest to largest. Write your answers in your notebook.

a) $-5, 4, 5, -7, -3, 2, 0$

b) $-9, -3, -7, -5, 1, -12, -1$

c) $-5, -8, -3, -9, -1, -4, -10$

d) $5, -3, -7, 4, 2, -6, -11, 12$

20 Find these absolute values.

a) $|+12|$

b) $|-7|$

c) $|-8|$

d) $|+23|$

e) $|-432|$

f) $|-12|$

21 Decide if these statements are *true* or *false*.

a) Two numbers can have the same absolute value.

b) There's a number with a negative absolute value.

c) There's a number with an absolute value of 0.

d) There's a number whose absolute value cannot be calculated.

22 Find these opposites.

a) op(−14)

b) op(+99)

c) op(+35)

d) op(−101)

e) op(−43)

f) op(−35)

23 Find these opposites and absolute values.

a) op(op(−12))

b) op(op(+12))

c) |op(+12)|

d) |op(−12)|

e) op(|−12|)

f) op(|+12|)

24 Copy and complete using $>$ or $<$.

a) op(−3) ☐ |−4|

b) op(|−17|) ☐ op(+15)

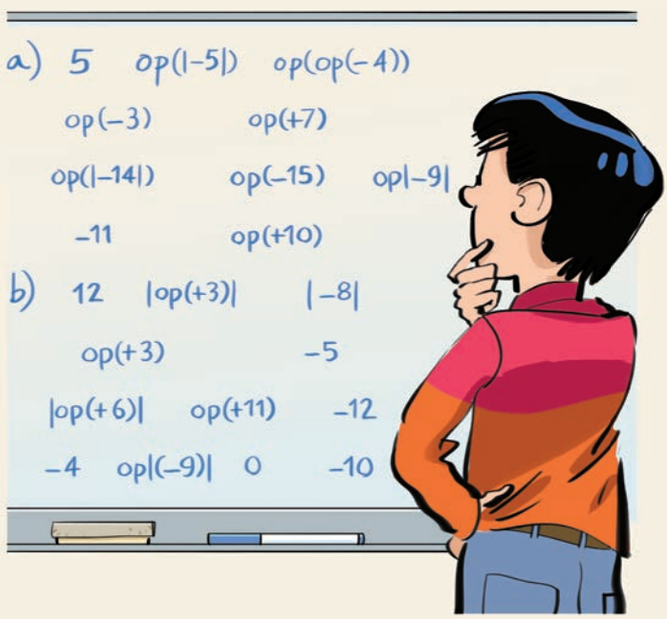
c) op|−12| ☐ −11

d) −(op(+15)) ☐ |−17|

e) |−10| ☐ op(+11)

f) −15 ☐ op(op(−14))

25 Order these numbers from largest to smallest.



26 Write all the integers with these characteristics.

a) Their absolute value is less than 3.

b) Their opposite is greater than 3 and less than 6.

c) Their absolute value is greater than −3.

d) Their absolute value is less than −3

e) Their opposite is less than −5 and more than −9.

27 Write two numbers between:

a) |−9| and op(+2)

b) |+6| and op(−3)

c) op(+3) and |+3|

d) op(−5) and |−5|

CLIL zone

28 Listen to the students describe a number and choose the correct option.

a) −9 b) 3 c) −5 d) 9 e) −7 f) 5

Take note

When doing operations with integers, always look at the sign in front of the brackets.

- If there is a + sign in front of the brackets, the number stays the same.

$$3 + (-5) = 3 - 5$$
$$3 + (+5) = 3 + 5$$

- If there is a − sign in front of the brackets, the number is replaced by its opposite.

$$3 - (-5) = 3 + 5$$
$$3 - (+5) = 3 - 5$$

3 Addition and subtraction of integers

Look at the information on the screen. What temperature is expected in 4 days' time?

The current temperature is −5 °C.
We must add:

- a positive number if the temperature increases (↑).
- a negative number if the temperature decreases (↓).

We do the operations in the order in which they appear.

$(-5) + (+6) + (+2) + (-4) + (-5) = -5 + 6 + 2 - 4 - 5$

$= 1 + 2 - 4 - 5 = 3 - 4 - 5 = -1 - 5 = -6$

In 4 days' time, the temperature will be −6 °C.

This is how we **add two integers**.

- If they have the same sign, add the numerical parts. The result will have the same sign as the numbers.
  For example: $3 + 5 = 8$ $(-3) + (-5) = -8$
- If one number is positive and the other number is negative, subtract the numerical part of the negative number from the numerical part of the positive number. The result will have the sign of the number with the larger numerical part.
  For example: $-3 + 5 = 2$ $3 + (-5) = -2$

To **subtract two integers**, add the first number to the opposite of the second.

Take note

To multiply or divide integers, use the sign rule.

$+ \cdot + = +$	$+ : + = +$
$+ \cdot - = -$	$+ : - = -$
$- \cdot + = -$	$- : + = -$
$- \cdot - = +$	$- : - = +$

4 Multiplication and division of integers

7 litres of water per minute is leaking from a water tank. How many litres will leak out of the tank in 5 minutes?

- We can use the integer −7 to represent 7 *litres of water per minute is leaking*.
- We can calculate how many litres of water leak out in 5 minutes by multiplying −7 by 5.

To do this:

1. multiply the numerical parts.
2. the numbers have different signs, so the result is negative:

$$(-7) \cdot 5 = -35$$

35 litres of water will leak out of the tank in 5 minutes.

This is how we **multiply or divide two integers**.

1. Multiply or divide the numerical parts.
2. Apply the **sign rule** to the result. Use a plus sign (+) if the numbers have the same sign, and a minus sign (−) if they have different signs.

Activities

Worked example

29 Calculate: $-(-2)+(-7)+3-(-6)$

Solution

Do the operations in the order in which they appear, remembering to look at the signs before the brackets.

$$-(-2)+(-7)+3-(-6)=2-7+3+6=4$$

30 Do these additions and subtractions. Write your answers in your notebook.

a) $(+12)+(+18)$
b) $(+12)+(-18)$
c) $(-7)+(+9)$
d) $(-12)+(-18)$
e) $(+8)-(+11)$
f) $(+14)-(-7)$
g) $(-12)-(+18)$
h) $(-12)-(-18)$

31 Calculate.

a) $3-5$
b) $12-9$
c) $-5+15$
d) $-12-5$
e) $-17+12$
f) $-15-9$

Worked example

32 Calculate: $3+(-5)-(4-7)-(12-5)$

Solution

$3+(-5)-\underline{(4-7)}-\underline{(12-5)}$ — Do the brackets first.

$=3+(-5)-(-3)-7=3-5+3-7=-6$

33 Calculate.

a) $-12+(15-20)+3-(-8)$
b) $3-(5-7)-(4+(-7)-3)+1$
c) $-(5+(-12))+(5-9+(-1))$
d) $-22-(-7+(17-9))+(13-8)$

34 Copy and complete.

a) $-7+\square=5-8$
b) $-14+\square=3+(-19)$

35 Joan has 235 euros in his bank account. He needs to pay three bills: one for 195 euros, one for 73 euros and one for 45 euros. How much will he have left in his account after he has paid the bills?

36 Copy and complete with the missing sign.

a) $4\cdot(\square 5)=-20$
b) $(-16):8=\square 2$
c) $(-12):(-4)=\square 3$
d) $3\cdot(\square 7)=-21$

37 What sign will the result have?

a) $12\cdot(-35):(-7)\cdot(-450)$
b) $(-45)\cdot(-58)\cdot(-135):(-30)$

38 Calculate.

a) $14\cdot(-4)$
b) $(-7)\cdot 7$
c) $(-42):(-7)$
d) $12\cdot(-3):4$

39 Copy and complete with the missing integer.

a) $50:\square=(-25)$
b) $\square\cdot(-12)=96$
c) $\square:(-6)=(-15)$
d) $\square\cdot(-6)=72$

40 Calculate.

a) $|-3|\cdot(-5)$
b) $|-5|\cdot \text{op}(-3)$
c) $(-2)\cdot|(-15):5|$
d) $\text{op}(18:(-2))\cdot(-3)$

41 Write this as a calculation and find the result.

I spend €8 on lunch every day. How much do I spend on lunch in a week?

42 Úrsula has a bank account that she only uses to repay a loan. Every month, her bank statement shows a transaction for −125 euros, which is her loan repayment of 125 euros.

a) By how much will her balance have changed after one year?
b) If her balance has decreased by 875 euros, how many monthly payments has she made?

43 The product of two integers is 18. What could the two integers be?

The product of three integers is 19. What could those integers be?

CLIL zone

44 Work with a classmate. Write an operation using additions, subtractions, multiplications and divisions. Don't show it to your partner. Say your operation to your partner using the phrases below. Your operation can have as many steps as you like. Your partner works out the answer. If your partner is correct, swap roles. If your partner is incorrect, discuss any mistakes and then swap roles.

I start with the number... *I add/subtract...* *I multiply/divide by...*

5 Powers and square roots

Powers with a negative base

Charo knows that she can write a multiplication of several equal factors in the form of a power.

When written as a power, the factor is called the base. The number of times the factor is repeated is called the exponent.

$$(-3)\cdot(-3)\cdot(-3)\cdot(-3)\cdot(-3)=(-3)^5$$

To calculate the result, we use the sign rule for the multiplication of integers:

$$(-3)^5=\underbrace{(-3)\cdot(-3)}_{(-3)^2=+9}\cdot\underbrace{(-3)\cdot(-3)}_{(-3)^2=+9}\cdot(-3)=\underbrace{9\cdot 9}_{+\,\cdot\,+\,=\,+}\cdot(-3)=\underbrace{81\cdot(-3)}_{+\,\cdot\,-\,=\,-}=-243$$

When a negative number is raised to an even power, the result is a positive number. When a negative number is raised to an odd power, the result is a negative number.

Take note

positive result

$(-3)^6 = 3^6$

negative base positive base

$(-3)^7 = -3^7$

negative result

The value of a **power with a negative base** depends on whether the exponent is odd or even.

- If the **exponent** is **even**, the value is the opposite of the base raised to the exponent. The result is positive.

$$(-a)^{even}=a^{even}$$

- If the **exponent** is **odd**, the value is the opposite of the base raised to the exponent, with a negative sign.

$$(-a)^{odd}=-a^{odd}$$

Square root of an integer

Charo also knows that the square root of a positive number is another number that, when multiplied by itself, gives the original number. For example:

The square root of 25 is 5 because $5^2 = 25$.

The square root of 25 is also -5 because:

$$(-5)^2=25$$

25 has two square roots: 5 and -5.

To calculate the square root of a negative number we need to find a number that, when multiplied by itself, gives the original negative number.

For example, what number multiplied by itself gives -25?

$$(?)^2 = -25$$

We know already that the value of any number raised to an even power is always positive. It isn't therefore possible to calculate square roots of negative numbers.

- All positive integers have two square roots: a positive one and a negative one.
- It isn't possible to calculate the square root of a negative integer.

Activities

45 Calculate. Write your answers in your notebook.

a) $(-2)^3$
b) $(-2)^6$
c) $(-2)^2$
d) $(-2)^7$
e) $(-2)^5$
f) $(-2)^4$

46 Are the results positive or negative?

a) $(-32)^{13}$
b) 314^{46}
c) $(-31)^{481}$
d) $(-91)^{132}$
e) $(-25)^{43}$
f) 428^{35}

47 Convert to powers with positive bases.

a) $(-15)^{13}$
b) $(-105)^{32}$
c) $(-23)^{53}$
d) $(-8)^{37}$
e) $(-42)^{46}$
f) $(-37)^{98}$

48 Write these products as powers with positive bases.

a) $2 \cdot (-2) \cdot (-2) \cdot 2$
b) $3 \cdot (-3)$
c) $(-7) \cdot (-7) \cdot 7 \cdot (-7)$
d) $(-5) \cdot 5 \cdot 5 \cdot 5 \cdot 5$
e) $(-9) \cdot 9 \cdot (-9) \cdot (-9) \cdot (-9)$
f) $(-10) \cdot 10 \cdot 10 \cdot (-10) \cdot (-10)$

49 In each pair of powers, are the two results the same?

a) $(-3)^4$ and -3^4
b) -7^{13} and $(-7)^{13}$
c) $(-5)^{12}$ and 5^{12}
d) $(-2)^3$ and -2^3
e) 9^{45} and $(-9)^{45}$
f) 11^{11} and $(-11)^{11}$
g) 13^5 y $(-13)^5$
h) -19^6 y $(-19)^6$

50 How many square roots do these numbers have?

a) −3 481
b) 11 025
c) 8 649
e) −23 716
f) −40 401
g) 103 041

51 What are the square roots of these numbers? Use the information provided.

a) 529 if $529 = 23^2$
b) 2 209 if $2\,209 = 47^2$
c) 676 if $676 = 26^2$

52 Calculate the square roots of these numbers.

a) 16
b) −25
c) −4
d) 81
e) 49
f) −10 000
g) −32
h) 100

Mathematical language

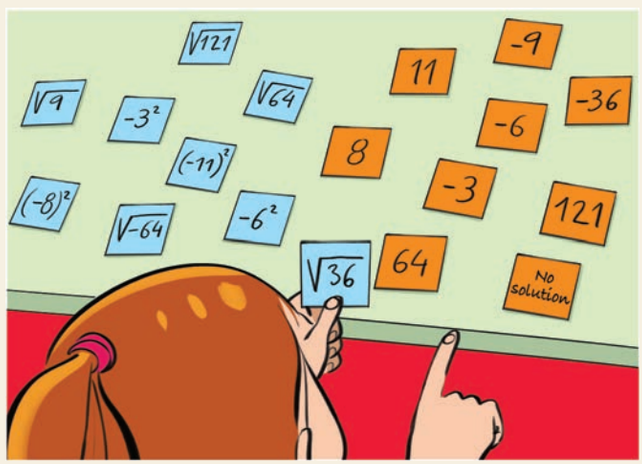
The square root symbol is: $\sqrt{\ }$

$$\sqrt{a} = b \rightarrow b^2 = a$$

53 Find the values of these square roots.

a) $-\sqrt{625}$
b) $-\sqrt{361}$
c) $\sqrt{529}$
d) $-\sqrt{1156}$
e) $\sqrt{784}$
f) $-\sqrt{1225}$

54 Copy and match the expressions to the numbers.



CLIL zone

55 Work with a classmate. Take it in turns to choose a number from the list and use the phrases below to say a true statement about your number. Your classmate says what your number is and explains how they worked it out.

a) −8 b) 9 c) −27 d) 5 e) 8 f) −9 g) −5 h) 27

My number is a square root of... *My number is ... raised to the power of...*
My number is ... squared *My number is positive/negative.*

6 Operations with powers

Powers with the same exponent

Multiplying powers with the same exponent

$5^3 \cdot (-2)^3 = (5 \cdot 5 \cdot 5) \cdot ((-2) \cdot (-2) \cdot (-2))$

$= (5 \cdot (-2)) \cdot (5 \cdot (-2)) \cdot (5 \cdot (-2)) = (-10) \cdot (-10) \cdot (-10) = (-10)^3 = -10^3$

Multiply the bases and keep the exponent as it is.

Dividing powers with the same exponent

$(-6)^2 : (-2)^2 = ((-6) \cdot (-6)) : ((-2) \cdot (-2))$

$= ((-6) : (-2)) \cdot ((-6) : (-2)) = 3 \cdot 3 = 3^2$

Divide the bases and keep the exponent as it is.

Take note

$a^n \cdot b^n = (a \cdot b)^n$

$a^n : b^n = (a : b)^n$

Powers with bases of the same absolute value

To **multiply or divide powers with bases of the same absolute value**, first write all the powers with a positive base, preceded by a minus sign if necessary. Then apply the rules for powers of natural numbers.

$(-5)^2 \cdot 5^4 = 5^2 \cdot 5^4 = 5^{2+4} = 5^6$

$3^5 \cdot (-3)^3 = -(3^3 \cdot 3^5) = -(3^{3+5}) = -3^8$

$(-7)^6 : 7^2 = 7^6 : 7^2 = 7^{6-2} = 7^4$

$2^8 : (-2)^5 = -(2^8 : 2^5) = -(2^{8-5}) = -2^3$

Remember

Operations with powers of natural numbers

- To **multiply or divide powers with the same base**, keep the base as it is and add or subtract the exponents.

  $a^p \cdot a^q = a^{p+q}$ $a^p : a^q = a^{p-q}$

- To **raise a power to a power**, keep the same base and multiply the exponents.

  $(a^p)^q = a^{p \cdot q}$

Power of a power

To **raise a power to a power**, first apply the properties of powers with a natural number as a base. Then, if the base is negative, write the power with a positive base.

$(4^3)^2 = 4^{3 \cdot 2} = 4^6$

$(4^5)^3 = 4^{5 \cdot 3} = 4^{15}$

$((-3)^3)^2 = (-3)^{3 \cdot 2} = (-3)^6 = 3^6$

$((-3)^5)^3 = (-3)^{5 \cdot 3} = (-3)^{15} = -3^{15}$

- **To multiply or divide powers with the same exponent**, keep the exponent as it is, and multiply or divide the base numbers. If the base is negative, write the power with a positive base.
- To **multiply or divide powers with bases of the same absolute value**, write all the powers with the positive base preceded by a minus sign if necessary, then add or subtract the exponents.
- To **raise a power to a power**, keep the same base and multiply the exponents. Then write as a power with a positive base.

Powers with an exponent of 0 or 1

$(-8) : (-8) = 1$

$(-2)^3 : (-2)^3 = (-2)^0 = 1$

$(-2)^{3-3} = (-2)^0$

$16 : (-8) = -2$

$(-2)^4 : (-2)^3 = (-2)^1 = -2$

$(-2)^{4-3} = (-2)^1$

Take note

$a^0 = 1$ $a^1 = a$

- A **power with an exponent of 0** is equal to 1.
- A **power with an exponent of 1** is equal to the base.

Activities

56 In your notebook, write as single powers.

a) $3^5 \cdot 2^5$ c) $12^7 : 4^7$

b) $7^3 \cdot 3^3$ d) $32^2 : 16^2$

57 Write as single powers.

a) $(-2)^4 \cdot 6^4$ c) $18^7 : (-2)^7$

b) $(-3)^5 \cdot (-5)^5$ d) $(-24)^6 : (-6)^6$

58 Write as single powers with positive bases.

a) $(-2)^5 \cdot 6^5$ d) $(-12)^3 : (-6)^3$

b) $(-2)^5 \cdot (-6)^5$ e) $12^3 : 6^3$

c) $2^5 \cdot (-6)^5$ f) $(-12)^3 : 6^3$

Worked example

59 Write as a single power with a positive base.

$$(-2)^4 \cdot 6^4 : (-3)^4 \cdot (-5)^4$$

Solution

Do the operations from left to right.

$(-2)^4 \cdot 6^4 : (-3)^4 \cdot (-5)^4 = (-12)^4 : (-3)^4 \cdot (-5)^4$

$= 4^4 \cdot (-5)^4 = (-20)^4 = 20^4$

60 Calculate, giving each answer as single power with a positive base.

a) $8^2 \cdot (-3^2) \cdot 2^2 : 6^2$

b) $(-12)^5 : (-3)^5 \cdot 5^5 : 2^5$

c) $3^{10} \cdot (-4)^{10} : (-2)^{10} : (-3)^{10}$

61 Write as single powers with positive bases.

a) $2^5 \times (-2)^3$ c) $(-5)^8 \cdot 5^3$ e) $4^7 \cdot (-4)^2$

b) $(-7)^8 \times 7^2$ d) $6^{12} \cdot (-6)^5$ f) $3^6 \cdot (-3)^4$

62 Write as single powers with positive bases.

a) $2^5 : (-2)^3$ d) $6^{12} : (-6)^5$

b) $(-7)^8 : 7^2$ e) $4^7 : (-4)^2$

c) $(-5)^8 : 5^3$ f) $3^6 : (-3)^4$

63 Write as single powers with positive bases.

a) $\left((-5)^3\right)^3$ c) $\left((-9)^3\right)^8$ e) $\left((-3)^3\right)^7$

b) $\left((-3)^8\right)^2$ d) $\left((-2)^2\right)^5$ f) $\left((-8)^6\right)^4$

Worked example

64 Write as a single power with a positive base.

$$3^2 \cdot (-3)^5 : (-3)^2$$

Solution

The bases of the powers are not equal, but they have the same absolute value, so we write them as powers with positive bases.

$3^2 \cdot (-3)^5 : (-3)^2 = 3^2 \cdot (-3^5) : 3^2 =$

Negative base and odd exponent

Negative base and even exponent

If you multiply a positive number by a negative number, the result is negative.

$= -3^2 \cdot 3^5 : 3^2 = -3^7 : 3^2 = -3^5$

If you divide a negative number by a positive number, the result is negative.

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65 Write as single powers with positive bases.

a) $3^5 \cdot 3^4 : (-3)^3$ e) $(-3)^2 \cdot 3^4 \cdot 3^5$

b) $(-11)^8 : 11^2 \cdot (-11)^5$ f) $(-5)^6 : (-5)^4 \cdot 5^5$

c) $(-7)^9 : 7^3 : 7^2$ g) $(-2)^5 : 2^3 \cdot (-2)^7$

d) $9^2 \cdot (-9)^7 : (-9)^5$ h) $5^8 : (-5)^3 : (-5)^2$

66 Write as single powers with positive bases.

a) $2^5 \cdot (-2)^3 : 2^5 \cdot \left((-2)^3\right)^2$

b) $(-5)^9 \quad (-5)^3 : \left(5^2 \cdot (-5)^3\right)^2$

c) $\left((-3)^5\right)^3 : \left((-3)^3\right)^2 \cdot 3^2$

d) $7^5 : \left((-7)^3 : 7^2\right)^4 \cdot (-7)^3$

67 Calculate these powers.

a) $(-3)^2$ b) 3^2 c) -3^2 d) $(-3)^0$ e) -3^0 f) 3^0

68 Write as single powers with positive bases.

a) $3^4 \cdot (-3) : (-3)^2$ d) $\left(3^4 : (-3)^3\right)^5 \cdot (-3)$

b) $(-5)^5 : (-5) : 5^2$ e) $(-7)^3 \cdot \left((-7)^3 : 7\right)^4$

c) $\left((-2)^4 : 2^4\right)^2 \cdot (-2)$ f) $\left(2^5 : (-2)\right) \cdot \left((-2)^3\right)^3$

CLIL zone

69 Listen to the students expressing these as single powers with positive bases. Correct their mistakes.

a) $4^2 \cdot (-3)^2$ b) $5^6 : (-5)^3$ c) $(3^1)^3$ d) $(-2)^5 : (-2)^3$

7 Combined operations

Operations without brackets

As with natural numbers, when we do combined operations with integers we must follow the correct order of operations.

$$12 - 6 \cdot 2^2 : 3 - (-5) - \sqrt{25} \cdot 3$$

1. Calculate all the powers and roots.
2. Do all the multiplications and divisions. If there are several of these, work from left to right.
3. Do all the additions and subtractions. If there are several of these, work from left to right again.

$$12 - 6 \cdot \mathbf{2^2} : 3 - (-5) - \mathbf{\sqrt{25}} \cdot 3$$
$$= 12 - \mathbf{6 \cdot 4 : 3} - (-5) - \mathbf{5 \cdot 3}$$
$$= 12 - 8 - \mathbf{(-5)} - 15$$
$$= \mathbf{12 - 8} + 5 - 15$$
$$= \mathbf{4 + 5} - 15 = \mathbf{9 - 15} = -6$$

Take note

When working with square roots, the $\sqrt{\ }$ symbol refers to the positive root.

$$\sqrt{49} = 7$$

Remember

When doing combined operations with integers, follow the order of operations:

1. brackets.
2. powers and roots.
3. multiplication and division (from left to right).
4. addition and subtraction (from left to right).

Operations with brackets

If an expression with integers includes operations in brackets, we must also follow the correct order of operations.

$$(-3)^2 + 5 \cdot (4 - 6) + \sqrt{16} - 1$$

1. Do all the operations in brackets.
2. Calculate all the powers and roots.
3. Do all the multiplications and divisions. If there are several of these, work from left to right.
4. Do all the additions and subtractions, working from left to right.

$$(-3)^2 + 5 \cdot (4 - 6) + \mathbf{\sqrt{16}} - 1$$
$$= \mathbf{(-3)^2} + 5 \cdot (-2) + \mathbf{\sqrt{16}} - 1$$
$$= 9 + \mathbf{5 \cdot (-2)} + 4 - 1$$
$$= \mathbf{9 - 10} + 4 - 1 = \mathbf{-1 + 4} - 1$$
$$= \mathbf{3 - 1} = 2$$

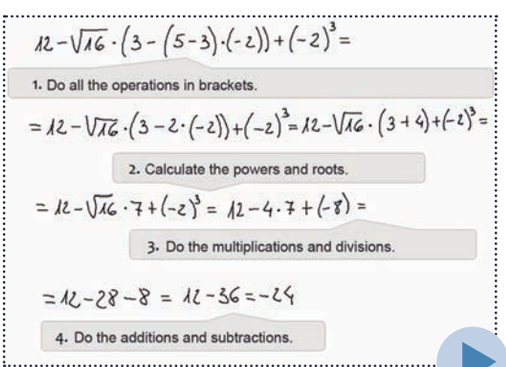
Worked example

70 **Evaluate this expression with combined operations.**

$$12 - \sqrt{16} \cdot (3 - (5 - 3) \cdot (-2)) + (-2)^3$$

Solution

Follow the correct order of operations.



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Lost in translation

In English we often use the acronym BIDMAS to remember the correct order of operations:

Brackets
Indices
Division and **M**ultiplication
Addition and **S**ubtraction

Activities

71 Calculate, following the order of operations. Write your answers in your notebook.

a) $7-(5-8)+3-4\cdot 2$

b) $5-13+(5-7+1)-(3-6)$

c) $-12-(4-12)+(5-7)+1$

72 Calculate.

a) $5-3\cdot 2+7$

b) $2\cdot(-5)+3\cdot(-2)$

c) $-4\cdot(-3)+5$

d) $5\cdot 3+(-2)\cdot 4$

73 These operations have been calculated incorrectly. Find the mistakes and correct them.

a) $3\cdot(5-7)+8=3\cdot 2+8=6+8=13$

b) $4-5\cdot 2+(-3)=(-1)\cdot 2+(-3)$
$=-2+(-3)=-5$

c) $-3+4\cdot(-3)-(-5)=-3+12-(-5)$
$=-3+12+5=14$

d) $18-(-9):(-3)+1=18+3+1=22$

74 Calculate.

a) $3-4\cdot 5+3\cdot(-3)$

b) $-7\cdot(-2)-3\cdot 4+2$

c) $12-15:(-3)+(-4)\cdot 5$

d) $-7-3\cdot(-5)+4-3\cdot(-5)$

75 Copy and complete these operations, putting the brackets in the correct places.

a) $3-5\cdot 4+1=-22$

b) $2-5\cdot 3+5=-4$

c) $3\cdot 4+1-7=8$

d) $3-5+3\cdot 2=-13$

76 Calculate.

a) $13-4\cdot(-3)^2$

b) $4\cdot(-5)+(-5)^2$

c) $2^3-3\cdot(-2)^3$

d) $5^2\cdot 2+(-2)^3\cdot 3$

77 Calculate.

a) $3-(4-3\cdot 2)+4\cdot(-3)+1$

b) $-5+12:(6-4\cdot 2)-(-3)$

c) $7\cdot(-2)+(6-3\cdot 4)+(-5)$

d) $-(3-7)+(12-3\cdot 6)-16:(-4)$

78 Do these combined operations.

a) $4\cdot(-3)+(-2)^5:4-7$

b) $-12+(-2)^3\cdot\sqrt{9}-(-5)$

c) $7+\sqrt{16}\cdot(-3)-5+3^2$

d) $(-3)^3-(-5)\cdot\sqrt{36}+5\cdot(-3)$

79 Find the results in each case.

a) $1-4\cdot\left(3-(-2)^2\right)+5$

b) $-(4-7)+7^2-24:\left(-\sqrt{64}\right)$

c) $4-5^2:(2-7)+\sqrt{16}$

d) $(-3)^2+5\cdot\left(3^2-4^2\right)-\sqrt{25}$

80 Calculate.

a) $3-\left((-2)^3-4\cdot(-5)\right)-\left(-\sqrt{49}\right)$

b) $15+\left(18:(-6)-4\cdot\sqrt{4}\right)+(-3)^3$

c) $-12+\left(5+18:\left(-3^2\right)\right)+\sqrt{25}$

81 Calculate. Pay attention to the brackets.

a) $5-\left(3\cdot\left(3^2-4\cdot\sqrt{25}\right)-12\right)+8:\left(-\sqrt{16}\right)$

b) $6\cdot(-3)+\left(12+7\cdot\left(\left(-\sqrt{9}\right)-2\right)-3\right)+2^2$

c) $4-(5+(-9))-\left((4-7)\cdot\sqrt{36}\right)$

82 Alicia puts 125 euros into her bank account each month. If there are currently 875 euros in the account, write mathematical expressions for the following situations:

a) The number of months the account has been open.

b) The money in the account three months ago.

83 The metric system uses the Celsius scale to measure temperature. However, in the United States the temperature is still measured in degrees Fahrenheit. The formulas to convert from one scale to the other are:

$$F=1{,}8\cdot C+32 \quad \text{or} \quad C=\frac{F-32}{1{,}8}$$

Use these formulas to convert −20 °C to degrees Fahrenheit and to convert 14 °F to degrees Celsius.

CLIL zone

84 Listen to the student describing the order of operations. Make a note of any mistake. Then use the phrases below to write a description of how to do combined operations using the correct order of operations.

To do combined operations, first we...
additions/subtractions/divisions/multiplications

Then we do/calculate...
roots/powers

Next we do/calculate...
brackets