



MYP Mathematics

A concept-based approach



Year
1

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2

Percentages

In this unit your use and understanding of percentages will give you a glimpse into the experience of refugees. However, the study of percentages could also be the gateway to discoveries in other contexts on a personal and global scale.



Personal and cultural expression

Entrepreneurship

Do you have an idea for your own business? What would it take to start your own company? What characteristics make for a successful entrepreneur? Answering these questions and interpreting the answers requires an understanding of percentages.

- Some research studies have shown that 90% of new companies fail within the first five years. Why does this happen?
- A start-up loan from the bank will charge interest, which is a percentage of the total amount. Could your new business afford to repay the interest?



Building a successful company also means understanding how to run a business. How do you know what price to charge for the product? How can you increase your profits without increasing the price? Percentages will help you analyze these questions and find optimal solutions.



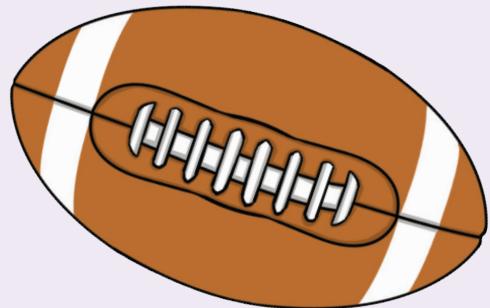
Globalization and sustainability

Population and demography

Demography is the study of human populations. It can show us how the human population is broken down into different segments and how those segments change over time.

What is the current population of the planet? What was it fifty years ago? What percentage is male? Female? Teenage? Employed? Interested in American football? Religious? Likes pizza? Researchers look into different questions, for different reasons.

While some of these questions may not seem very important, looking at trends in the human population can help us to understand how the world might look in the future and explain differences between humans in different regions. The results of these questions may even point out where inequities exist.



2

Percentages

Inequality and difference

KEY CONCEPT: FORM



Related concepts: Equivalence and Quantity

Global context

In this unit you will work with percentages, fractions and decimals to explore the global context of **fairness and development**. You will apply the relationships between percentages, fractions and decimals as you explore worldwide refugee displacement, working conditions, nutrition, and the availability of safe drinking water. You will also have opportunities to reflect on and discuss what it might be like to be a refugee in a foreign country.

Statement of Inquiry

Inequality and difference become clearer through the use of equivalent forms of quantities.

Objectives

- Representing a number in different forms – fractions, decimals and percentages
- Converting between equivalent forms of numbers – fractions, decimals and percentages
- Calculating percentage increase and decrease
- Applying mathematical strategies to solve problems involving percentages

Inquiry questions

- F** What are different forms of numbers?
When are two things equal?
- C** How can different forms be equivalent?
When is it beneficial to use different forms?
- D** Can fairness be calculated?

ATL1 Collaboration skills

Practise empathy

ATL2 Organization skills

Plan short- and long-term assignments; meet deadlines

You should already know how to:

- say and write decimal numbers
- round numbers correctly
- reduce fractions to their simplest form
- mentally multiply and divide by powers of ten
- find missing values in equivalent fractions



Introducing percentages

Life seems to be full of percentages. They appear in advertising, in the news, even on our devices. But what do they mean?

Why do we use them? Do they only describe a quantity or amount, or can they be used in other ways? Is it possible for percentages to help you learn important lessons, such as what it might be like to live in someone else's shoes?



Being comfortable with percentages will not only help you in your daily life, but may also help you understand someone else's life a little better.

Reflect and discuss 1

Answer these questions individually before discussing them as a class.

- What do you think the word *percent* means?
- What is the definition of the word *cent*?
- In the following two examples, what does the word *per* represent?
 - A bike courier works 7.5 hours per day.
 - A cruise ship travels at 30 nautical miles per hour.
- Putting these together, what do you think the word *percent* means?



Percentages

A percent, or percentage, is a ratio which can be written as a fraction out of one hundred. The symbol used to show a percentage is: %. For example, 20% is read as ‘twenty percent’, meaning twenty out of one hundred.



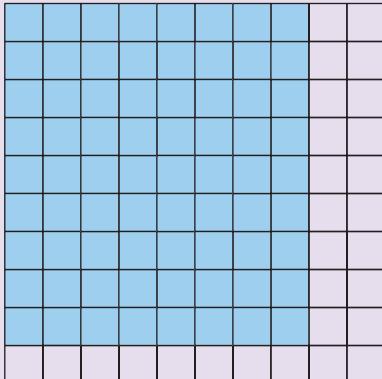
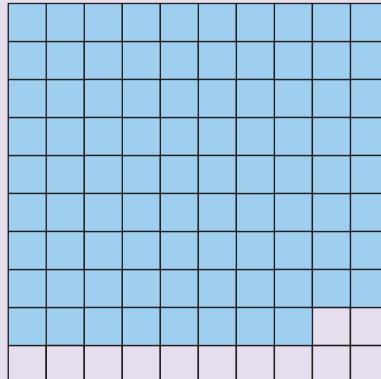
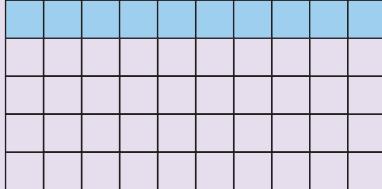
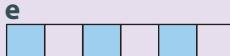
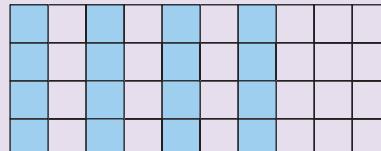
Activity 1 – Uses of percentages

Percentages are a very useful way of expressing ratios and fractions. In pairs, find 10 real-life examples of where percentages are commonly used. You will score 1 point if several pairs have the same example, 3 points if only one other pair has it, and 5 points if yours is the only pair that has it.

The most common way to represent a percentage is by using notation, e.g. 75% or 13.5%. However, percentages can also be represented using models. These models can make percentages easier to understand by giving them a visual representation.

Activity 2 – Visual representation of percentages

- 1 Write down the percentage represented by the shaded section of each grid (e.g. 60%).

a**b****c****d****e****f**

- 2** Describe how you were able to identify the percentage for the grids that do not have 100 squares.
- 3** Create two of your own grids to represent 80%.
- 4** Create two of your own grids to represent 30%.
- 5** Make a creative and unusual visual representation of 50%.

Reflect and discuss 2

- Can you have a percentage that is larger than 100%? If so, how would you represent it visually?
- If you could have a percentage greater than 100%, what would it mean? Give an example to explain your reasoning.

In Activity 2, you saw that a percentage can be represented visually using shaded squares. Although using 100 squares makes representing and identifying percentages much simpler, percentages can also be shown using a different number of squares. Even though percentages are quantities or ratios written out of 100, you can sometimes simplify these ratios to produce equivalent fractions. In fact, percentages and fractions are linked because they can be equivalent forms of the same quantity.

Investigation 1 – Key percentages

- 1 You have seen that a percentage is a ratio out of 100. Some percentages, like 50%, are easy to calculate since they are easy to simplify (in this case, one-half). Based on this idea, which percentages are easy to simplify? Copy the grid below and make a list of them. At the same time, divide the grid into the correct proportions, as shown in the first two examples.



100%			
50%		50%	
25%	25%	25%	25%

► Continued on next page

- 2 If you can calculate the percentages in step 1, multiples of these should also be fairly easy to calculate. Copy the table on the right and make a list of these other percentages. If there is more than one way to calculate a percentage, list the easiest way. An example has been done for you.
- 3 Based on the investigation, state a general rule for finding any percentage.
- 4 Verify your rule for other percentages that you have not looked at yet.

Percentage	How you can calculate it
75%	multiply 25% by 3

Reflect and discuss 3

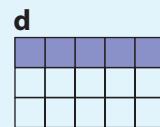
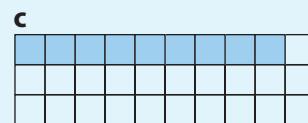
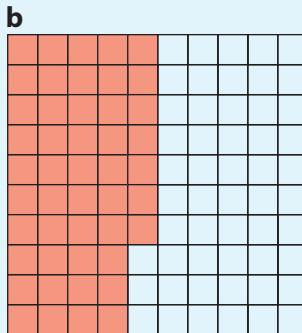
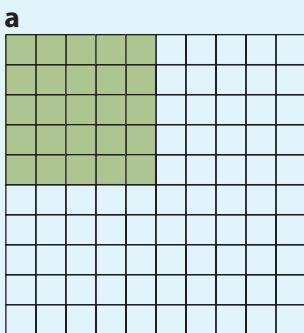
ATL1

- What makes some percentages easier to calculate than others?
- In some parts of the world, when people eat at restaurants, a common practice is to tip the server 10%, 15% or 20% of the cost of the meal. Can you think of an easy way to calculate these tip amounts?
- Why do you think servers receive tips? Suppose you ate at a restaurant, but you didn't know about the tipping custom and therefore you left no tip. How would it feel to be your server? What questions would your server ask himself/herself?



Practice 1

- 1 Write down the percentage represented by the shaded section of each grid.



- 2 For each percentage below, draw two grids of different sizes and then shade the percentage.

a 60% b 20% c 80% d 12% e 4%

- 3 If 10% of a certain quantity is 12, find the following percentages of the same quantity.

a 20% b 30% c 70% d 100% e 5% f 25%

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- 4** If you knew what 4%, 5% and 6% of a certain quantity was, how could you calculate the following percentages of the same quantity?
- a** 12% **b** 9% **c** 10% **d** 30% **e** 11% **f** 14% **g** 17%
- 5** In 2015, 6% of the people forced to flee their homes were hosted by countries in Europe. This represented approximately 4 million people.
- a** Using this data, approximate the number of people represented by the following percentages.
- i** 12% of the people fled to the Americas.
ii 30% fled to an African country (not including North Africa).
iii 15% fled to a country in the Asia-Pacific region.
iv 39% fled to a country in the Middle East or North Africa.
- ATL1** **b** Imagine you had to leave your country because of a civil war. What would it feel like to move to a new place where you don't know anyone and don't speak the language? How would it feel to know that you may never go back home? What other feelings would you have?
- 6** As of January, 2017, Canada had taken in approximately 40 000 Syrian refugees. Approximately 15 000 of these refugees were sponsored privately by individual Canadians who formed small groups of citizens to finance the refugees' first year in Canada. What percentage of the refugees were sponsored by private citizens?

Equivalent forms

Percentages and fractions

A percentage is a fraction out of 100, and any fraction can be converted to a percentage.

Activity 3 – Percentages and fractions

- 1** Convert each percentage to a fraction out of 100.

64%

8%

19%

120%

250%

- 2** Convert each fraction to a percentage.

$\frac{32}{100}$

$\frac{8}{100}$

$\frac{71}{100}$

$\frac{135}{100}$

$\frac{135}{200}$

Being able to convert between equivalent forms is a very important skill in mathematics. When converting fractions to percentages, it is easiest to convert the quantity to an equivalent fraction out of 100 and then write the number as a percentage.

Example 1

Q Convert $\frac{19}{25}$ to a percentage.

A $\frac{19}{25} = \frac{\square}{100}$

$$\frac{19 \times 4}{25 \times 4} = \frac{76}{100}$$

$$\frac{76}{100} = 76\%$$

Therefore, $\frac{19}{25} = 76\%$.

Multiply the denominator by 4 to make an equivalent fraction out of 100.
Multiply the numerator by the same amount in order to maintain equality.

Write the fraction out of 100 as a percentage (since 'percent' means 'out of 100').

Reflect and discuss 4

What do you think you would do to convert $\frac{4}{80}$ to a percentage, since its denominator is not a factor of 100?

Example 2

Q Write $\frac{7}{35}$ as a percentage.

A $\frac{7 \div 7}{35 \div 7} = \frac{1}{5}$

$$\frac{1 \times 20}{5 \times 20} = \frac{20}{100}$$

$$\frac{20}{100} = 20\%$$

Therefore, $\frac{7}{35}$ as a percentage is 20%.

Simplify the fraction.

Rewrite as an equivalent fraction out of 100.

Rewrite the fraction as a percentage.

Reflect and discuss 5

Sometimes the denominator of a fraction does not divide evenly into 100, yet you'd still like to convert it to a percentage.

- How could you use a calculator to help you find the percentage represented by $\frac{3}{17}$?
- Write down the steps you could use with your calculator to convert a fraction to a percentage.
- If a calculator is not available, what other option might you have? Use this method and round your answer to the nearest hundredth.

Activity 4 – Refugees around the world



Research the statistics needed below and use a calculator to help you answer these questions. (Try unhcr.org as a statistical resource website.)

- 1 What is the approximate world population?
- 2 What is the approximate world refugee population?
- 3 What percentage of the world population consists of refugees? Round your answer to the nearest tenth.
- 4 Roughly 3.5 billion people have wealth of less than US \$10 000. What percentage of the global population does this represent? Round your answer to the nearest hundredth.
- 5 Approximately 50% of all refugees are children (under the age of 18). How many refugees are children? Round your answer to the nearest whole number.



Reflect and discuss 6

ATL1

- What would it mean to have a total wealth of less than US \$10 000? What would life be like? How might your priorities be different than they are now?
- In Activity 4, why are some numbers rounded to one decimal place, but others to the nearest whole number?



Practice 2

1 Convert these fractions to percentages.

a $\frac{1}{2}$ b $\frac{17}{20}$ c $\frac{7}{10}$ d $\frac{3}{5}$ e $\frac{31}{50}$

f $\frac{3}{4}$ g $\frac{11}{25}$ h $\frac{3}{20}$ i $\frac{25}{25}$ j $\frac{47}{50}$

2 Convert these fractions to percentages.

a $\frac{6}{12}$ b $\frac{12}{30}$ c $\frac{6}{40}$ d $\frac{7}{70}$ e $\frac{18}{45}$

f $\frac{21}{28}$ g $\frac{36}{60}$ h $\frac{81}{90}$ i $\frac{24}{32}$ j $\frac{9}{15}$

3 Convert these fractions to percentages. Use two significant figures.

a $\frac{5}{18}$ b $\frac{2}{21}$ c $\frac{11}{43}$ d $\frac{71}{89}$ e $\frac{18}{19}$

f $\frac{24}{53}$ g $\frac{33}{92}$ h $\frac{50}{61}$ i $\frac{21}{17}$ j $\frac{1}{11}$

4 At least $\frac{4}{5}$ of humanity live on \$10 per day or less. What percentage of the population does this represent?

5 Of the estimated 2 billion children living in the developing world, approximately 400 million do not have access to safe drinking water.

Approximately what percentage of children in developing countries do not have safe water to drink?



Activity 5 – The world's resources

Are the world's resources divided evenly? By looking at the population of each continent/region and its gross domestic product (GDP), you can get an idea of the distribution of the world's resources.

1 Referring to the table on the next page, research the current population and GDP of each region and fill in the appropriate columns in a copy of the table.

2 Calculate each region's share of the global GDP as a percentage and fill in the second column.

3 Calculate the percentage of the world population that each region's population represents and fill in the fourth column.

Gross domestic product, or GDP, is a measure of the income of individuals and companies in a region.

- Based on the percentage breakdown of the population, divide your class into groups that accurately represent the six regions. Fill in the last column with the number of students representing each region.
- The world's GDP can be represented by 100 counters. Divide up the counters between the regions, based on the GDP percentage breakdown in the table. Give each group of students the counters for their region.
- Take the counters for your region and divide them evenly among the group members. What do you notice? Make a general statement about the world's allocation of resources.
- What if the counters were replaced with chocolate coins and each group member was allowed to eat them? Would that be fair?

Continent or region	GDP	Percentage of global GDP	Population	Percentage of global population	Class size breakdown
Africa					
Asia					
Europe					
North America					
Oceania					
South America					
Total GDP:		Total Population:			

Reflect and discuss 7

- Are the world's resources distributed evenly? Justify your answer.
- Do you think they should be distributed more equitably? If so, what can be done to make the distribution more equitable?
- As you have discovered, the world's resources are not distributed evenly. How would you feel if you were a citizen of a country that received a small percentage of the world's resources? How would you feel if you were a citizen of a country that received a large percentage of the world's resources?

equitable: fair and impartial



Percentages and decimals

A decimal can be converted to a percentage using a process that you are going to determine in the investigation below.



Investigation 2 – Percentages and decimals



Copy the table below and complete it to discover the process of converting a decimal to a percentage.

	0.45	0.3	0.295
Say the number (out loud) based on its place value and write it down.			
Write this number as a fraction.			
Convert to an equivalent fraction out of 100, if necessary.			
Write the fraction as a percentage.			

- 1 How would you explain the process of converting a decimal to a percentage to a student who was absent for this lesson? Demonstrate and verify the process with two other decimal numbers.
 - 2 How would you convert a percentage to a decimal?

Reflect and discuss 8

- How does our decimal number system help with the process of converting decimals to percentages?
 - How can you tell if a percentage and a decimal are equivalent?

Example 3

- Q** Represent each percentage as a decimal.

a 17%

b 62.5%

A a $\frac{17}{100} = 17 \div 100$

$$17 \div 100 = 0.17$$

Think of the fraction as a division.

b $\frac{62.5}{100} = 62.5 \div 100$

Move the digits over two places when dividing by 100.

$$62.5 \div 100 = 0.625$$

Think of the line between the numerator and denominator as a division sign.

$$62.5 \div 100 = 0.\underline{6}25$$

Move the numbers over two places when dividing by 100.

You know how to convert fractions to terminating decimals (decimals that have a *finite* number of decimal places), but there are actually two other kinds of decimals:

- repeating decimals – decimals that have a sequence of digits that repeat *infinitely*
- non-terminating, non-repeating decimals – decimals that do not terminate and do not repeat. These are also called *irrational numbers* and you typically need a calculator to convert them between equivalent forms.



Investigation 3 – Repeating decimals

You will look at repeating decimals to ensure you can recognize these numbers in both decimal and fractional form.



A repeating decimal is often indicated with a line over the top of the digits to be repeated.

For example: $0.\overline{6} = 0.6262626262\dots$ $1.\overline{534} = 1.5343434\dots$ $70.\overline{137} = 70.137137137\dots$

In the table below, the first column contains terminating decimals and their equivalent fractions, and the second column contains repeating decimals and their equivalent fractions.

Terminating decimals	Repeating decimals
$0.7 = \frac{7}{10}$	$0.\bar{7} = \frac{7}{9}$
$0.1 = \frac{1}{10}$	$0.\bar{1} = \frac{1}{9}$
$0.23 = \frac{23}{100}$	$0.\overline{23} = \frac{23}{99}$
$0.017 = \frac{17}{1000}$	$0.\overline{017} = \frac{17}{999}$

- Looking at the denominators, what is the key difference between the fraction equivalents of terminating and repeating decimals?
- Focusing on the denominators again, how do you know how many digits will be repeating infinitely?
- What are the steps to convert a repeating decimal to a fraction?
- Take a look at the following:

$$0.\bar{3} = \frac{1}{3} \quad 0.\bar{6} = \frac{2}{3} \quad 0.\overline{72} = \frac{8}{11}$$

Can you follow the same steps to convert the repeating decimals in these examples to fractions? Explain.

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- 5 Decide which of the following conversions are correct. For any that are incorrect, identify how to fix them so that the decimals and fractions are equivalent.

a $0.4 = \frac{4}{9}$

b $0.\overline{35} = \frac{35}{99}$

c $0.\overline{011} = \frac{11}{999}$

d $0.\overline{2} = \frac{2}{11}$

e $0.\overline{24} = \frac{8}{33}$

f $0.\overline{36} = \frac{4}{11}$

g $0.\overline{06} = \frac{3}{50}$

h $0.\overline{375} = \frac{375}{9}$

i $0.\overline{17} = \frac{7}{40}$

It is the opposite process to convert a fraction to a repeating decimal.

Example 4

Q Convert $\frac{50}{99}$ to a decimal.

Determine the number of 9s in the denominator; this is the number of repeating digits in the decimal.

A 99 – so two digits repeat

NOTE: if the denominator does not contain only 9s you may have to convert the fraction first. If the denominator cannot be converted to contain only 9, then it is not a repeating decimal.

0.50

The numerator is placed in those decimal places.

0. $\overline{50}$

Draw a line over the top of the decimal to represent the infinite repeating of those digits.

$$\frac{50}{99} = 0.\overline{50}$$

To convert repeating decimals to percentages, it is best to simply look at the number in decimal form and convert using the process you already know (multiply by 100). You will typically round the number to the most appropriate number of decimal places, given the context of the question.

In Example 4, the repeating decimal was $0.\overline{50}$ so this would be 50.5050%. This number would round to 51%, accurate to the nearest percentage, and 50.5% if you wanted the number accurate to the nearest tenth.



Practice 3

1 Convert each decimal to a fraction and then a percentage.

a $0.87 = \frac{\square}{\square} = \square\%$ b $0.5 = \frac{\square}{\square} = \square\%$ c $0.732 = \frac{\square}{\square} = \square\%$

d $0.09 = \frac{\square}{\square} = \square\%$ e $0.006 = \frac{\square}{\square} = \square\%$ f $1.2 = \frac{\square}{\square} = \square\%$

2 Convert each percentage to a decimal.

a 12% b 46% c 103% d 38.5% e 8.3% f 12.25%

3 Write these numbers in all three forms: fraction, decimal and percentage.

Write each fraction in its simplest form.

a $\frac{2}{25}$

b 0.8

c 35%

d $\frac{4}{9}$

4 Which is the largest number in each of the following groups?

Justify your answer.

a $\frac{3}{10}$ or 28% or 0.28

b $\frac{17}{25}$ or 0.7% or 0.7

c 1.03 or 100%

d $\frac{3}{15}$ or $\frac{2}{9}$

e $\frac{24}{33}$ or 70% or 0.67

f $\frac{8}{40}$ or 20% or 0.2

5 Which numbers are equivalent in each of these sets?

a $\frac{42}{150}$

16%

0.28

b $\frac{17}{34}$

4%

0.4

$\frac{21}{60}$

28%

0.286

$\frac{104}{260}$

44%

0.40

6 Order each set of numbers from smallest to largest.

a $\frac{2}{5}$

36%

0.6

$\frac{2}{11}$

14%

0.186

b $\frac{1}{9}$

9%

0.16

$\frac{7}{20}$

21%

0.05

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- 7 Order each set of numbers from largest to smallest.

a $\frac{31}{33}$, 87%, 0.89, $\frac{45}{50}$, 91%, 0.98

b $\frac{21}{70}$, 27%, 0.09, $\frac{11}{55}$, 4%, $0.\overline{136}$

- 8 In each of these scenarios, which option offers the greatest amount?

Justify each answer using mathematics.

a 25% of one bag of popcorn, or 10% of three of the same sized bags of popcorn

b 50% of two mangos, or 40% of three of the same sized mangos

c 30% of one energy bar, or 5% of six of the same sized energy bars.

- 9 The International Labour Organization (ILO) estimates that 60% of the 250 million children around the world who work in sweatshops live in Asia. Approximately how many Asian children work in sweatshops?

sweatshop: a workshop, often in the clothing industry, employing workers at low wages, for long hours, and under poor and often unsafe conditions

- 10 The average hourly wage of a woman making clothes in Bangladesh is the equivalent of US \$0.13. The United States Federal minimum wage as of January 2017 was US \$7.25 per hour. Approximately what percentage of the US minimum wage is the Bangladeshi clothes worker's wage?

- 11 In India, approximately 5% to 30% of the 340 million children under 16 are classified as child laborers. About how many children does this represent?

For 11, give your answer as a range.

- 12 The charity Oxfam is working to help refugees in Bangladesh. In September 2017, there were 470 000 Rohingya Muslim people from Myanmar living as refugees in Bangladesh. Only 50% of these refugees had access to clean drinking water.

a How many people does this percentage represent?

So far, Oxfam has been able to provide clean drinking water to 100 000 of the refugees who did not previously have access to clean drinking water.

b What percentage of the total number of people now have clean water?

Reflect and discuss 9

- Which form – percentage, decimal or fraction – do you prefer when ordering numbers?
- What are the advantages and disadvantages of each form? Make a list for each one.
- How are fractions, decimals and percentages equivalent forms?

Activity 6 – Conversion bingo

Draw a 4 by 4 grid and fill it in with a combination of fractions, decimals and percentages, at least four of each. Here is an example.

Your teacher will give you the specific guidelines for what numbers you can put in your grid. (For example, you may be told that you can use any percentages that are a multiple of 10. So you must come up with decimals, fractions and percentages that as a percentage are a multiple of 10.) You cannot include two equivalent numbers in different forms (e.g. 20% and 0.2).

Once you have written the numbers in your grid, the game will begin. Your teacher will call out a number in one of the three forms and you can cross it off your grid if you have that number in any of the three forms. The winner is the first student to get four in a row, column, or major diagonal.

20%	0.4	30%	$\frac{17}{20}$
$\frac{1}{4}$	0.75	0.05	45%
10%	16%	$\frac{1}{15}$	90%
$\frac{6}{35}$	70%	0.68	$\frac{3}{5}$

Formative assessment – Food for refugees

The United Nations High Commissioner for Refugees (UNHCR) recommends that a refugee should consume at least 2100 calories per day. Yet in many refugee camps, refugees get less than this. For example, in one Tanzanian refugee camp, refugees were given only about 1400 calories per day.

criteria
C, D

- 1 What percentage of the minimum daily calories were the refugees in the Tanzanian camp given?
- 2 In many refugee camps, refugees end up trading their food for other non-food supplies that they need. If a refugee was eating only a third of the minimum daily calorie recommendation, how many calories per day would that be?



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The table shows approximately what a Syrian refugee might eat *per day* in a Jordanian refugee camp. Remember that the UNHCR recommends that a person should have at least 2100 calories daily.

Food	Grams (approx.)	Calories (approx.)	Type of nutrient
rice	274	1100	carbohydrate
flour	57	230	carbohydrate
lentils	24	97	protein
dried chickpeas	12	49	protein
sardines	18	71	protein
canned kidney beans	57	229	protein
vegetable oil	43	386	fat



- 3 What is the number of total daily calories in this refugee diet?
- 4 What percentage of a refugee's calories come from protein?
- 5 What percentage of a refugee's calories come from carbohydrates?
- 6 What percentage of a refugee's calories come from fat?
- 7 Make a similar list of the foods that you eat in a typical day.
- 8 Using an online source, calculate the approximate number of calories for every item on your list and state the type of nutrient.
- 9 Calculate the total number of calories you consume in one day. Calculate how many calories you consume daily of fat, protein, and carbohydrates.
- 10 What percentage of your calories come from each of these three nutrient groups?
- 11 Explain the degree of accuracy of your results in step 10. Describe whether or not your results in step 10 make sense.
- 12 How does your diet differ from the refugee diet? How is it similar? (Compare the number of calories, breakdown of food groups, etc.)
- 13 How do you think refugees feel, having to follow this diet? Explain.

ATL1



Applications of percentages

There are many practical applications involving percentages which you will see on a regular basis, such as finding the discount on an item for sale in a store or calculating the tax when purchasing an item. Remembering that percentages and fractions are really just equivalent forms can help you to solve these types of problems.

Example 5

Q a What percentage of 80 is 8?

b What is 30% of 25?

c 50% of what number is 13?

A a 80 is the whole and 8 is the part, so the fraction is $\frac{8}{80}$.

$$\frac{8}{80} = \frac{8 \div 8}{80 \div 8} = \frac{1}{10}$$

$$= \frac{1 \times 10}{10 \times 10} = \frac{10}{100} = 10\%$$

Therefore, 8 is 10% of 80.

Convert the fraction to a percentage.

b $\frac{30}{100} = \frac{\square}{25}$

$$\frac{30 \div 4}{100 \div 4} = \frac{7.5}{25}$$

Therefore, 30% of 25 is 7.5.

In this case, 25 is the whole and the part is unknown.
We need to find the equivalent to the fraction represented by 30%.

Since $100 \div 4 = 25$, we need to divide the numerator by 4 as well.

c $\frac{50}{100} = \frac{13}{\square}$

$$\frac{1}{2} = \frac{13}{\square}$$

$$\frac{1 \times 13}{2 \times 13} = \frac{13}{26}$$

Therefore, 50% of 26 is 13.

In this case, 13 is the part but the whole is unknown.
We need to find the equivalent to the fraction represented by 50%.

Simplify the first fraction.

Reflect and discuss 10

- How would you write 30% as a decimal?
- What does the word 'of' mean in mathematics?
- What is another way of solving problems like 'Find 70% of 25'?

Practice 4

1 Find the percentage in each of the following amounts.

a What percentage of 100 is 75?

c What percentage of 300 is 75?

e What percentage of 50 is 18?

g What percentage of 96 is 24?

b What percentage of 48 is 36?

d What percentage of 620 is 124?

f What percentage of 220 is 132?

h What percentage of 850 is 340?

► Continued on next page



2 Find the number when:

- | | |
|-----------------------------------|------------------------------------|
| a 50% of the number is 20 | b 20% of the number is 8 |
| c 10% of the number is 12 | d 25% of the number is 38 |
| e 80% of the number is 208 | f 14% of the number is 49 |
| g 62% of the number is 124 | h 130% of the number is 78. |

3 Find the number when given the percentage.

- | | | |
|---------------------|----------------------|---------------------|
| a 25% of 100 | b 40% of 100 | c 20% of 50 |
| d 80% of 100 | e 10% of 1000 | f 70% of 200 |
| g 15% of 200 | h 5% of 60 | i 100% of 80 |
| j 35% of 200 | k 8% of 50 | l 12% of 32 |
| m 60% of 55 | n 45% of 80 | o 90% of 20 |

4 In a store, an item has a new price of \$63 after a reduction of 30%. What was the original price?

5 In Spain, the average annual income of a male is approximately €26 000 and the average annual female income is approximately €20 000. Approximately what percentage of the average male income is the average female income?

6 In Canada, the average annual income of a female aged 25 to 34 is 77% of a male in the same age group. If the average annual income of a female is approximately \$50 000, how much, rounded to the nearest thousand dollars, does the average male in the group earn?

7 In Australia, the average full-time weekly salary for a woman is approximately 84% of the full-time weekly salary for a man.

If an Australian man earns approximately \$1600 per week, how much does an Australian woman earn?



ATL1 **8** What feelings would you experience if you earned less than someone else for doing the same work? Explain.

9 During a six-week period in 2012, the organization Doctors Without Borders managed to vaccinate approximately 75% of a population at imminent risk of a disease called cholera in Guinea and Sierra Leone, averting a potential disaster. What was the total population if 170 000 people were vaccinated?

Many countries around the world add a sales tax (sometimes called a value added tax) onto most of the goods that are sold. In some countries, the tax is included in the listed price, while in others the tax is added on to the listed price. To make sure you have enough money, you need to be able to quickly determine what that final price will be. You can do this easily in your head without a calculator.



Activity 7 – More than one way

You want to buy an item that costs £60. However, a tax of 15% will be added. What is the amount of the tax?

One way to solve this is to divide the problem in two:

$$15\% = 10\% + 5\%$$

To find 10% of 60, you can multiply 0.1×60 .

$$0.1 \times 60 = 6$$

Since 5% is half of 10%, find half of £6, which is £3.

Therefore, the total tax is £6 + £3 = £9.

- a Solve the problem by using one calculation involving a decimal.
- b Solve the problem by using two equivalent fractions.
- c Solve the problem in as many other ways as you can think of.

Reflect and discuss 11

- Which method did you prefer to use when solving the problem in Activity 7?
- Will your choice of method depend on the question, or will you always use the same one?
- How can different methods produce equivalent results?



Practice 5

- 1 a** Calculate the tax on each item, then calculate the total cost of each item. Use the average sales tax worldwide, which is 15%.



- b** You have \$100 to spend. Can you afford the painting (including sales tax)? Explain.
- c** List three items you could buy (including sales tax) with your \$100. Write down their total cost.
- d** What combination of items can you buy that is the closest to your \$100, without going over it? Remember to include sales tax on each item.
- 2 a** Find the price of each of the following items after a discount is applied (often called the ‘sale price’).
- i A shirt that was originally \$62 is discounted by 50%.
 - ii A pair of \$120 shoes are now marked 30% off.
 - iii A \$40 concert ticket now has 40% off.
- b** What is the total price of each item in part **a** if the worldwide sales tax of 15% is applied after the discount?
- 3** Carlos buys a backpack for \$30. There is a 10% discount followed by a 15% tax. How much will the backpack cost in total?
- 4** The war in Syria, which led to millions of refugees, also increased prices of goods dramatically.
- a** A shawarma sandwich used to cost 100 Syrian pounds (SYP), but has now increased by 100%. What is the new price of the sandwich?
 - b** A tax of 10% was then added by the government on the shawarma sandwich. What is the new price of the sandwich?

You can solve these problems in a similar way to the tax problems, but this time subtracting the discount from the original price.



► Continued on next page

- ATL1**
- c The price of six eggs was 120 SYP and increased by 550%. What is the new price of six eggs?
 - d The price of a kilogram of butter used to be 520 SYP. However, that has increased by 250%. What is the new price of a kilogram of butter?
 - e Butter could be replaced by margarine, the cost of which is only 350 SYP per kilogram. Using the new price of butter, what percentage of the price of a kilogram of butter is this?
 - f Imagine you are living in Syria and the price of food is increasing as you have seen. How would it feel to be the head of a family trying to provide for them, despite your salary not increasing? What would you consider as your options?

Percentage change

Often, we are interested in finding by what percentage a quantity changes, which is a comparison of a change in value to the original value.

The percentage change is found by:

$$\text{Percentage increase or decrease} = \frac{\text{new amount} - \text{original amount}}{\text{original amount}} \times 100\%$$

Example 6

- Q**
- a In 2008, the number of undernourished people was 1 billion worldwide. In 2010, approximately 900 million people were undernourished globally. Find the percentage decrease.
 - b Despite the scenario highlighted in part a, in 2008, the United States wasted 126 billion pounds of food. In 2010, that amount climbed to 133 billion pounds. Find the percentage increase, rounded to the nearest hundredth.
- A**
- a To find the percentage change (decrease):

$$\frac{900\,000\,000 - 1\,000\,000\,000}{1\,000\,000\,000} \times 100\% = -10\%$$

It represents a 10% decrease.

Use the percentage change formula.
The negative value here indicates a decrease.

- b To find the percentage change (increase):

$$\frac{133\,000\,000\,000 - 126\,000\,000\,000}{126\,000\,000\,000} \times 100\% \approx 5.56\%$$

It represents a 5.56% increase.

Give your answer to the degree of accuracy stated in the question.



Practice 6



- Calculate the percentage change of the following examples, clearly stating if it is an increase or a decrease. Round to 1 decimal place if necessary.
 - The percentage change from 270 to 60
 - The percentage change from 34 to 91
 - The percentage change from 580 to 2479
 - The percentage change from 8452 to 7203
 - The percentage change from 54 to 540
 - The percentage change from 3160 to 718
- There are currently approximately 3 400 000 births per year by girls who are under 17 years of age in sub-Saharan Africa and south and west Asia. It is predicted that the number would drop to approximately 3 100 000 births if all girls in those areas were to receive a primary school education. Calculate the predicted percentage decrease in births if all the girls receive a primary school education.
- It is predicted that the figure given in question 2 would drop even further, to 1 400 000 births, if all girls in those areas also had a secondary school education. Calculate the predicted percentage decrease in births if all the girls receive a secondary school education.



Search online for Math Playground Percent Shopping and play the game that involves calculating discounts and sale prices.

Shopping at Troy's Toys



Level 1
Find the discount and sale price.

Level 2
Find the percent discount.

Reflect and discuss 12

Discuss these issues regarding teenage pregnancies in small groups.

ATL1

- What would be the implications of such a drop in births? (Practice 6, question 3)
- Why do you think education would have such an impact on the number of births by girls under 17 years of age? How would it feel to be an educated female in sub-Saharan Africa or south and west Asia?
- Do you think that in developed countries 'education for all' is taken for granted?
- If you were to write a headline right now that captured the most important aspect of your discussion, what would that headline be?
- Share your headlines with the class.

Activity 8 – What the world eats

The quantity, quality and variety of foods that people eat around the world differ significantly between regions. You are going to look at the percentage breakdown of the daily diet of different countries and compare them to the world average.

Search online for ‘what the world eats’ to answer these questions:
<https://www.nationalgeographic.com/what-the-world-eats/>

World consumption changes

- 1 What was the average worldwide daily calorie intake in 1961?
- 2 What was it in 2011?
- 3 Calculate the percentage increase. Do you think this is a *significant* increase? Why do you think there has been an increase?
- 4 What category, as a percentage of the total, has increased the most over the years? Why do you think this is?



Your country compared to the world

- 1 Find the data for your country. If your country is not listed, your teacher will tell you a country that has a similar demographic.
- 2 Is your country’s calorie intake higher or lower than the world average? Calculate your country’s daily calorie intake as a percentage of the world average.
- 3 Discuss with a partner why your country’s calorie intake is higher or lower than the world average. Summarize three reasons to share with the class.
- 4 Look at the changes of your country’s calorie intake over the last 50 years. Pick two categories and calculate the percentage increase or decrease over the 50 years. What is significant about your calculations?



Your country compared to other countries

- 1 Look at the 22 countries listed on the right side of the webpage. Which country has the smallest calorie intake per day? Which has the highest? Calculate the percentage difference between the two. Give two explanations for why there is such a difference.
- 2 Find the country that has the highest percentage of each category in their daily diet. Create a table for your results. Did you find any of these surprising? Why?
- 3 Select two other countries that are different to your own country. Create a table to compare these countries in the way that you think would best allow the data to be compared. Record all necessary information in your table.
- 4 Based on their daily diet, how do you think the daily life of people in these countries differs from yours?

What will the future look like?

- 1 Calculate the average annual percentage change of daily calorie intake in a country of your choice.
- 2 In 20 years, if your country follows the same trend it has followed over the past 50 years, what will the daily calorie intake be?
- 3 In 20 years, what could be the health implications of this?



Unit summary

A *percent* or *percentage* is a ratio which can be represented as a fraction out of 100.

Fractions, decimals and percentages are all equivalent forms that express the same quantity.

Some percentages, like 50% or 25%, are easier to calculate because you know the fraction that they represent.

Conversions



A fraction to a percentage

Write an equivalent fraction out of 100. Then, write the number using the percent (%) symbol.

or

Divide the numerator by the denominator and then multiply by 100%.



A percentage to a fraction

Write the percentage as a fraction out of 100. Be sure to simplify the fraction whenever possible.



A decimal to a fraction

Say the decimal number using its place value (e.g. '35 hundredths') and then write the corresponding fraction. Be sure to simplify the fraction whenever possible.



A percentage to a decimal

Divide the percentage by 100 and write without the percent symbol.

Calculate percentage increase by dividing the amount of increase by the original quantity. This decimal can then be converted to a percentage by multiplying by 100%.

Calculate percentage decrease by dividing the amount of decrease by the original quantity. This decimal can then be converted to a percentage by multiplying by 100%.

Unit review



Launch additional digital resources for this chapter

Key to Unit review question levels:

Level 1–2

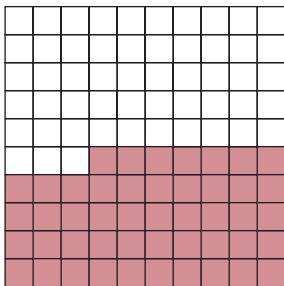
Level 3–4

Level 5–6

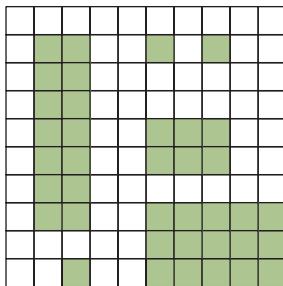
Level 7–8

- 1** Write down the percentage represented by the shaded section of each grid.

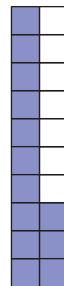
a



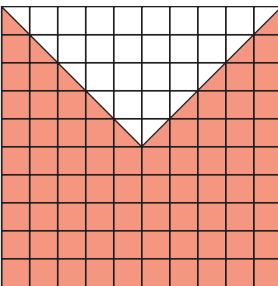
b



c



d



e



- 2** For each percentage below, draw two grids of different sizes and then shade the given percentage.

a 30% b 50% c 75% d 18% e 12%

- 3** If you knew what 3%, 8% and 10% of a quantity was, how could you calculate the following percentages of the same quantity?

a 9% b 11% c 21% d 18% e 16% f 26% g 77%

- 4** If 20% of a quantity is 30, find the following percentages of the same quantity.

a 10% b 40% c 100% d 30% e 4% f 24%

- 5** Convert these fractions to percentages.

a $\frac{9}{10}$	b $\frac{17}{20}$	c $\frac{14}{7}$	d $\frac{3}{12}$	e $\frac{22}{55}$
f $\frac{20}{50}$	g $\frac{12}{60}$	h $\frac{8}{99}$	i $\frac{37}{37}$	j $\frac{40}{11}$

- 6** Convert these fractions to percentages, and round to the nearest whole percentage.

a $\frac{3}{13}$	b $\frac{8}{35}$	c $\frac{14}{47}$	d $\frac{61}{66}$	e $\frac{7}{19}$
f $\frac{31}{51}$	g $\frac{12}{73}$	h $\frac{153}{88}$	i $\frac{36}{35}$	j $\frac{212}{100}$

7 Write down the following in all three forms: fraction, decimal and percentage. Give each fraction in its simplest form.

- a** 28% **b** 0.8 **c** $\frac{13}{20}$ **d** 0.32 **e** 55%
- f** $\frac{7}{15}$ **g** 92% **h** $\frac{4}{20}$ **i** $0.\overline{60}$ **j** 110%

8 Which is the largest number in each of the following lists?

Justify your answer.

- a** $\frac{4}{5}$ or 75% or 0.81 **b** 0.92 or 90% or $\frac{17}{19}$ **c** $\frac{2}{3}$ or 65% or 0.63
- d** $\frac{1}{4}$ or 26% or $0.\overline{25}$ **e** 0.55 or 53% or $\frac{6}{13}$

9 Which numbers are equivalent in each of the following sets?

- a** $\frac{12}{50}$, 25%, $\frac{1}{4}$, 0.24, $\frac{3}{8}$, 2.4%, 24%
- b** 5%, $\frac{2}{20}$, 0.5, $\frac{30}{600}$, 50%, $\frac{1}{2}$, 0.05

10 In the 2016 Olympics, the International Olympic Committee created a ‘Refugee Olympic Team’ to draw attention to the worldwide refugee crisis. Ten athletes were selected from four countries to take part in this team. Calculate the percentage of athletes from the following countries of origin.

- a** 5 athletes were originally from South Sudan.
- b** 2 athletes were originally from Syria.
- c** 1 athlete was originally from Ethiopia.

11 There are an estimated 700 million children living in the world who are of primary school age.

Approximately 70 million of them still do not have access to education. Approximately what percentage of primary-age children do not have access to education?

12 In the United States Senate, 20 senators out of 100 are women. In Canada, 88 women have been elected to the House of Commons out of 338 members. Which country has the higher percentage of women in its government house?



- 13** a What percentage of 50 is 10?
- b What is 30% of 20?
- c 15 is 45% of what number?
- d What is 60% of 90?
- e 40 is 25% of what number?
- f What percentage of 80 is 60?
- 14** Due to the efforts of many charitable organizations, enrollment in primary education has reached approximately 90% worldwide. If there are approximately 60 million children still not attending schools, approximately how many are enrolled in primary schools?
- 15** Many charitable organizations have worked hard to reduce child mortality. Since 1990, the number of deaths worldwide in children under five years old declined from approximately 13 million to approximately 6 million.
- a What is the approximate percentage decrease?
- b Assuming a constant rate of change, by approximately what percentage did the number of deaths drop per year?
- c If this trend continues at the same rate, what would you predict to be the number of deaths in children under five years old in ten years' time?
- 16** Approximately 780 million people in the world are illiterate. What percentage is that of the total population? (You found this statistic earlier in this unit). Of the 780 million, almost 70% are female. Approximately how many females are illiterate in the world?
- 17** a Before 2011, a typical Syrian could expect to live to be about 80 years old.
In 2014, during a civil war where millions of people died, life expectancy in Syria decreased to 56 years old. **Calculate** the percentage decrease in life expectancy.
- b Due to the civil war, 4 million refugees fled Syria. Some of these refugees fled to neighboring Jordan, where the population swelled from approximately 5 850 000 to approximately 6 500 000. **Calculate** the percentage increase in the population of Jordan.
- c Lebanon, another country that took in many refugees, saw its population increase from roughly 3 million to 4 million. **Calculate** the percentage increase in the population of Lebanon.

Summative assessment



'You are a refugee' assignment



You and your partner are refugees and have just arrived in your current city. You have no money, but the government gives you 'social assistance' funding to survive. Your teacher will tell you the monthly amount that you will receive. This amount needs to cover all living expenses. Your teacher will also tell you how many members are in your family and the ages of any children. You must find a place to live and prepare a monthly budget for rent, food, transportation, living expenses, clothes and other expenses. As a class, you will create a list of all of these expenses and every pair will use this list.

ATL2

This assignment has many different tasks that will require a lot of organization. You and your partner will need to collaborate well. It will be very important to set up a good organizational framework to ensure that you can both access all your information easily, and that it will not get lost.

Once you have read through the assignment with your partner, your first task will be to create a graphic organizer/diagram or other method to organize all of the different components in this assignment. The second task will be to create a to-do list along with a calendar of when each item will be completed.

Think about the following questions:

- How will you organize your information?
- How will you record and keep track of the websites you have used?
- How will you break up the tasks into manageable sections?
- How will you create a to-do list?
- What headings will your to-do list have?
- How will you communicate with each other?
- How will you manage your time so that you do not leave too much work to the last minute?

Once you have submitted this graphic organizer/diagram and to-do list, your teacher will provide you with the appropriate platform on a web-based application (connection through link or email) for you both to work on. Then you will set up the organization framework that you outlined. If technology cannot be used as the self-management tool for this assignment, you must still explain how you will organize and store your documents and information, and how you will manage your time.

Part A – Preparation

- 1 **Find a place to live.** You will look for an apartment in a local newspaper or online. Remember your budget and keep in mind the other expenses that you have! Once you have found your accommodation, locate it on a map of your city/town.

- 2 Find food to eat.** Write down a nutritious meal plan for breakfast, lunch and dinner for seven days. Be specific – don't just write 'sandwich' – consider all the parts of the sandwich you will need: bread, mustard, tomato, cheese, etc. Use a chart like the one below to record your meal plan.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Breakfast							
Lunch							
Dinner							
Snack							

a Once you have an idea of what you will eat in a week, you will have to calculate the cost of the items. First, create a grocery list for the food you will need to make the meals listed in your chart for one week.

Then put the list in a chart with three columns: the food item, the estimated cost and the actual cost. You will then estimate how much you think each item costs and calculate the total. You could create your chart using a spreadsheet so that you can do the calculations efficiently. To find the real costs, you can either search a grocery store's prices online, or go to the grocery store.

Consider a field trip to a supermarket as a class, or individually with your parents. Alternatively, use an online shop to find the costs of grocery items.

- b** Calculate the actual total cost of the groceries.
- c** Calculate the percentage difference between what you estimated the total cost of the groceries to be and their real cost.
- d** Explain the degree of accuracy of your results in part **c**. Describe whether or not your results make sense.

- 3 Money for other expenses:** You will set up a monthly budget spreadsheet. A budget tracks your income and your expenses to help you see how much money you will have left to save or how much you will be in debt. The budget is best created using a spreadsheet, so you can set up a formula for 'Income less total expenses'. This number should be positive so that you do not go into debt. Here is an example budget spreadsheet.

MONTHLY BUDGET	
CATEGORY:	Amount:
Income: (how much you earn)	
Expenses: (what you have to pay)	
Rent	
Groceries (multiply weekly amount by 4)	
[Create a row for each expense]	
Total expenses: (add up all expenses)	
Net Income: (Income less total expenses)	

- 4 As you complete this assignment, you will be recording your thoughts and reactions. Discuss what struggles you are facing, and your thoughts about how refugees might survive on this money. You will submit this reflection with your assignment.

Part B – Analysis

- 1 Using the grocery list based on your meal plan for the week, approximate what percentage of the grocery list is:
a carbohydrates **b** protein **c** fat
- 2 Compare your grocery list with your country's nutrition guidelines. How close does your grocery list match the recommended daily allowances of your country's nutrition guidelines?
- 3 Select one of the days of the week from Part A question 2 and calculate the number of calories one person will eat in a day as a refugee moving to a new country. To do this, divide the total by the number of people in your family. Compare this amount to what a person in a refugee camp eats in a day and what you actually eat in a day. (You calculated these in the formative assessment task earlier in the unit.)
 - a** Who consumes the greatest number of calories in a day?
 - b** Who consumes the greatest percentage of carbohydrates, protein and fat in a day: you, someone in a refugee camp, or the refugee moving to a new country?
 - c** Did any data in the comparison surprise you? Why or why not?
- 4 Looking at your monthly budget, calculate each expense as a percentage of the total. Did any of the expenses surprise you? Why or why not?

Reflect and discuss 13 – Empathy for a refugee

- Were you able to stay on budget? Describe any sacrifices you had to make.
- Were you able to eat nutritiously on the budget? Explain.
- How did the life you created as a refugee differ from the life you live now? What would it feel like to be a refugee? Explain your answers.
- What recommendations would you make to your city/country to make life better for refugees who move to your country?

ATL1

Self-assessment

ATL2

Self-management

Throughout this assignment, you have worked on developing your self-management skills to stay on task and meet the deadlines outlined by your teacher. Using a copy of the checklist below, reflect on how you were able to manage your organizational skills and affective skills throughout the assignment. In the first column, rank yourself as 1 (emergent), 2 (capable) or 3 (exemplary) to indicate how each skill has developed. In the next column, discuss what you did well and what you would do next time to improve each skill.

Skill	Emergent (1) Capable (2) Exemplary (3)	What would you do next time to improve this skill? If you think you developed this skill well, list the things you did that helped your development and would do again next time.
Managing time and tasks effectively		
Meet deadlines of the assignment <ul style="list-style-type: none">● Used a calendar effectively● Tasks were well spaced out – no cramming/rushing to complete tasks on time		
Create plans to prepare for summative assessments <ul style="list-style-type: none">● Created to-do lists, followed and updated them● Prioritized tasks on the to-do list● Broke down the assignment into manageable tasks		
Keep an organized and logical system of information files/notebooks <ul style="list-style-type: none">● Created a logical system to organize all documents/tasks in an effective way (e.g. using Google Docs effectively)● Used this system throughout the assignment so all information was easily accessible to both partners		
Select and use technology effectively and productively <ul style="list-style-type: none">● For organization and collaboration (Google Docs)● For researching information		
Managing state of mind		
Practise strategies to overcome distractions <ul style="list-style-type: none">● Turned off social media and email notifications while working on computer/tablet● Kept workspace tidy so resources were readily accessible		
Demonstrate persistence and perseverance <ul style="list-style-type: none">● Kept trying and revised plans when things did not go right● Learned from making mistakes instead of seeing them as setbacks		