UNIT 1 Natural numbers

Lost in translation

In Spanish, we write numbers greater than 999 with a dot or a half space for the thousands:

2.839 or 2 839

In English, we write these with a comma or with a half space:

2,839 or 2 839



Subtraction check

minuend

= subtrahend + difference

Division check

Dividend

= **d**ivisor \cdot **q**uotient + remainder

 $D = d \cdot q + r$

Lost in translation

We use the centred dot (\cdot) for horizontal multiplication in Spanish. In English, we sometimes use the centred dot, but the (\cdot) symbol is more common.

> 3 · 2 (English) 3 · 2 (Spanish)

Calculations with natural numbers

Addition, subtraction, multiplication and division

Ana has a shop. She buys newspapers from a wholesaler every day. At the end of each day, she returns all the unsold newspapers. This table shows a summary of the newspapers she bought and returned in one week. How many did she sell?



	Μ	Т	W	Т	F	Sa	Su
Bought	432	390	350	355	456	501	652
Returned	21	32	27	52	68	42	55

We can calculate the number sold each day and add the results together.

- Monday: 432 21 = 411
- Tuesday: 390 32 = 358
- Friday: 456 68 = 388
- Saturday: 501 − 42 = 459
- Wednesday: 350 27 = 323
- Thursday: 355 52 = 303
- Sunday: 652 55 = 597
- 411 + 358 + 323 + 303 + 388 + 459 + 597 = 2839 newspapers.

Addition is the total of two or more numbers combined. Subtraction is the opposite. It gives the difference between two numbers.

Manuel has stored his harvest of oranges in 137 boxes of 6 kg each. He decides to put the oranges into 8 kg boxes instead. How many boxes will he need? Will there be any oranges left over?

137	822 8
· 6	022 102
822	6

Manuel will have 102 boxes, with 6 kg left over.

- Multiplication is the repeated addition of the same number.
- Division is splitting a number into equal parts.

Properties of calculations with natural numbers

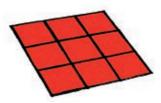
_	Addition	Multiplication			
Commutative	3 + 2 = 2 + 3	$3 \cdot 2 = 2 \cdot 3$			
Commutative	5 = 5	6 = 6			
	5 + (4 + 2) = (5 + 4) + 2	$5 \cdot (4 \cdot 2) = (5 \cdot 4) \cdot 2$			
Associative	5 + 6 = 9 + 2	$5 \cdot 8 = 20 \cdot 2$			
	11 = 11	40 = 40			
Neutral element	4 + 0 = 4	$4 \cdot 1 = 4$			
	$2 \cdot (5+3) = 2 \cdot 5 + 2 \cdot 3$				
Distributive	$2 \cdot 8 = 10 + 6$				
	16 = 16				

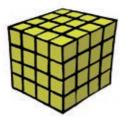
Write a number greater than 999 in your notebook. 😥 a) What's the order of the units, from largest to smallest? b) What's the value of each digit? 2 Do these additions. a) 32 + 4 506 + 294 c) 690 + 3 + 3491 + 14**b)** 562 + 3009 + 473d) 37 + 91 + 5 + 50563 Do these subtractions. a) 562 - 89c) 690 - 147 - 543b) 1295 - 453 d) 9001 - 17 - 3892>----Copy and complete these calculations. a) 256 + = 591+791 = 1005c) b) 48 + = 931d) +72 = 1235 Copy and complete these in your notebook. a) 432 - = 191 c) — 195 = 1 005 **b)** 927 - = 571d) = - 97 = 709 6 Do these multiplications. a) 701 · 5 c) 309 · 165 **b)** 23 · 45 d) 4 901 · 6 023 7 Find the quotient and the remainder for these divisions. a) 506:28 c) 5007:17 **b)** 2848:32 d) 64 368 : 596 8 Copy and complete the table, using the division check. Dividend Divisor Quotient Remainder 5 34 4 127 8 7 7 3 42 691 18 38 Check that these calculations balance. a) (23 + 12) + 7 = (12 + 23) + 7**b)** $32 \cdot (2 \cdot 12) = (2 \cdot 32) \cdot 12$ 🥲 What criteria did you use? have one tree for every 16 inhabitants?

ZODe

(1) Copy and complete, using the distributive property. a) $(9+3) = 4 \cdot (9+4)$ **b)** $7 \cdot (-+5) = -9 + -5$ Remember Finding the common factor is using the distributive property to change an addition or subtraction into a multiplication. – Worked example – 11 Find the common factor for these calculations. a) $3 \cdot 5 + 3 \cdot 9$ **b)** $8 \cdot 4 - 3 \cdot 4$ Solution a) $3 \cdot 5 + 3 \cdot 9 = 3 \cdot (5 + 9)$ **b)** $8 \cdot 4 - 3 \cdot 4 = (8 - 3) \cdot 4$ 12 Find the common factor for each calculation. a) $6 \cdot 4 + 6 \cdot 9$ c) $7 \cdot 8 - 5 \cdot 8$ d) $6 \cdot 9 + 4 \cdot 9$ b) $12 \cdot 10 - 12 \cdot 8$ Take note Every natural number can be written as the product of 1 and the number itself. For example: $5 = 5 \cdot 1 = 1 \cdot 5$ Find the common factor. a) $12 \cdot 5 - 3 \cdot 12 + 12 \cdot 8$ c) $3 \cdot 4 - 4 \cdot 2 + 4 \cdot 6$ b) $6 \cdot 9 - 6 \cdot 3 - 6 \cdot 2$ d) $7 \cdot 5 + 5 \cdot 9 - 4 \cdot 5$ Find the common factor. a) $4 \cdot 5 - 5$ c) $6 \cdot 5 - 6 + 6 \cdot 8$

- b) $9 + 2 \cdot 9$ d) $3 - 3 \cdot 2 + 4 \cdot 3$ 15 17 920 inhabitants live in a city. There's one tree for every 64 inhabitants. How many trees are there in the city? How many trees do they need to plant to
- 16 Listen to the news and answer the questions.
 - a) How many litres of water can each plane and each helicopter carry?
 - b) Calculate the total capacity of all the planes and helicopters mentioned.
 - c) What's the difference between the total capacity of the planes and the total capacity of the helicopters?







Press this button to calculate a number's square.



Press this button to calculate any power.

2 Powers of natural numbers

Lola uses small squares to make a larger square with 3 small squares on each side. How many small squares does she need to make the large square?

 $3 \cdot 3 = 3^2 = 9$ small squares

She uses small cubes to build a larger cube with 4 small cubes on each face.

How many small cubes does she need to build the large cube?

 $4 \cdot 4 \cdot 4 = 4^3 = 64$ small cubes

A **power** is a short way to write multiplying a number by itself. The **base** is the number we're multiplying. The **exponent** shows how many times we're multiplying it.

Exponent $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$ Base 5 times

This is how we write and read powers: $7^2 \rightarrow$ seven squared; $7^3 \rightarrow$ seven cubed; $7^4 \rightarrow$ seven raised to the power of four, and so on.

Squarc roots

Jorge has made a square mosaic using 16 equal-sized square tiles. How many tiles does each side of the mosaic have?

Find a number that, when multiplied by itself, equals 16.

$$4 \cdot 4 = 4^2 = 16$$

Therefore, each side of the mosaic has 4 tiles.

The **square root** of a number x is the number whose square is equal to x.

 $\sqrt{16} = 4$ because $4^2 = 16$ Radicand Root

Jorge takes 14 square tiles. He can't make a square mosaic with 14 tiles. There are two options:

- make a square mosaic with three square tiles per side, and have five tiles left over.
- make a square mosaic with four square tiles per side, but with two tiles missing.

 $3^2 = 9 < 14$

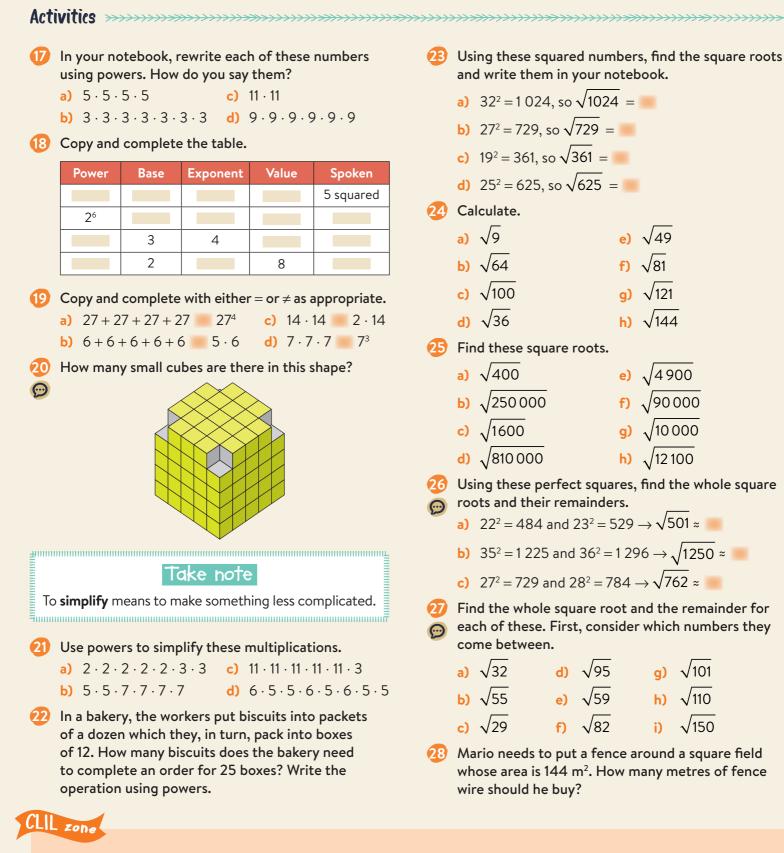
 $14 < 16 = 4^2$

Numbers that have an exact square root are called **perfect squares**. If the number isn't a perfect square, we find the **whole square root**. This is the highest perfect square which is lower than the number.

 $\sqrt{14} \approx 3$ because $3^2 = 9 < 14$ and $4^2 = 16 > 14$

The **remainder** is the difference between the radicand and the largest perfect square smaller than it.

Remainder: $14 - 3^2 = 5$



29 Source with a classmate. Use the phrases below to solve the problem. A cinema has 11 screens. Each screen has 11 rows and in each row there are 11 seats. What's the maximum capacity of the cinema? Express the answer as a power and calculate its value.

We need to divide/subtract/multiply/add ... to/from/by is how we express this as a power. Its value is...



When an equation or expression contains more than one mathematical operation, this is the order of the operations.

$$3 + 8 : 2 \cdot 3 - 4 \cdot 2 + 1$$

- First do the multiplications and divisions. If there's more than one, work from left to right.
- 2 Next, do the additions and subtractions. Again, if there's more than one, work from left to right. = 3 + 12 - 8 + 1 = 15 - 8 + 1= 7 + 1 = 8

Powers and roots

If powers and roots are also part of the equation or expression, do the operations in this order.

$$4 + 3 \cdot 2^2 - \sqrt{81}$$
 : 3

- 1 First do the powers and roots.
- 2 Then do the multiplications and divisions. If there's more than one, work from left to right.
- Finally, do the additions and subtractions. If there's more than one, work from left to right.

 $4 + 3 \cdot 2^{2} - \sqrt{81} : 3$ $= 4 + 3 \cdot 4 - 9 \cdot 3$ = 4 + 12 - 3

= **16** - **3** = 13

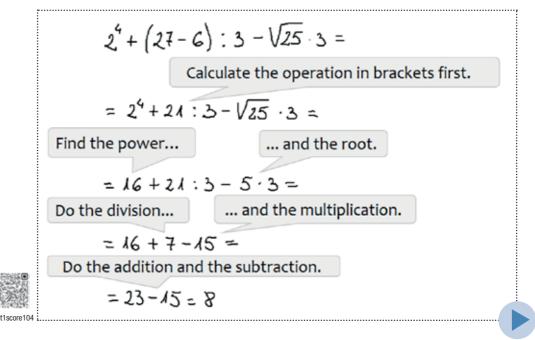
 $3 + 8 : 2 \cdot 3 - 4 \cdot 2 + 1$

 $=3+4 \cdot 3 - 8 + 1$

Brackets

If there are any operations inside brackets, do these first, then follow the same order as above.

$$2^4 + (27 - 6) : 3 - \sqrt{25} \cdot 3$$



Take note

The order for performing mathematical operations is universally accepted, so that there's never a disagreement over a calculation. The order is:

- 1 brackets.
- 2 powers and roots.
- 3 multiplications and divisions. If there's more than one, we work from left to right.
- **4** addition and subtraction. If there's more than one, we work from left to right.

30 Do these calculations in your notebook. d) 18 - 15 : 3 a) $4 + 7 \cdot 2$ **b)** $3 \cdot 7 - 8$ e) 45:9+7**c)** 8 – 10 : 2 f) 5+30:631 Do these calculations. a) $12 - 9:3 + 5 \cdot 4 - 7$ **b)** $32 + 12 - 12 \cdot 2 - 18 : 9$ c) $2 \cdot 4 - 15 : 5 + 10 - 3 \cdot 2$ d) $16 - 8 \cdot 2 + 22 : 11 + 7$ 32 Find the results of these combined operations. a) $3 \cdot 4 - 18 : 6 \cdot 2$ c) $1+4 \cdot 2 \cdot 3 - 2 \cdot 3$ **b)** $12 - 6 \cdot 3 : 2 + 1$ d) 12 - 8 : 4 + 30 : 6 : 533 Do these combined operations. a) $23 - 5 + 7 \cdot 3 - (25 - 9 - 5)$ **b)** 32:4-(8-5-1)-(12-11)-1c) $9 \cdot 5 + 15 : 3 - (32 - 3) - (18 - 8 - 1)$ d) $12 + 4 \cdot 6 - (4 + 10 - 7) - 2 + 8 : 4$ **34** Perform these calculations. a) $18:6+\sqrt{100} - 3^2$ c) $5^2 - \sqrt{64} + 4 \cdot 4 - 1$ **b)** $12 + 2^4 - \sqrt{36} + 5$ **d)** $44 : 4 + 3^3 - \sqrt{81} + 7 \cdot 2$ **35** Do these calculations. a) $9^2 - 5^2 \cdot 2 + \sqrt{4} \cdot 6 - 1$ **b)** $12 + \sqrt{36} : 3 + 2^3 - 5$ c) $48:6+66:\sqrt{121}-9$ d) $25 - \sqrt{25} \cdot 2 + 6^2 : 3 + 2^5$ 86 Perform these calculations. a) $12 + 4 \cdot 2 - (3 + 5) : 2$ b) $\sqrt{81+12:3+5\cdot(10-9+2)+2}$ c) $15 - 4 + 3 \cdot (12 - 4) : 6 + 7^2$ ZODe

- Work in a group of three to solve the problem.
 Some friends are having a party. They buy:
 4 lots of offer A
 7 lots of offer B
 6 lots of offer C
 - a) How many cans of each drink do they buy?
 - b) How much do they spend in total?
 - For cola/lemon/orange, the combined operation is ... The answer is...

Oo these calculations.

- a) $2 + 3 \cdot (5 + 4 \cdot 3) 1$
- **b)** $7+5-(14-5\cdot 2)+8:2$

- c) $25 + (9 4 \cdot 2 + 5) 12 : 6$
- → … Worked example " <<
- **38** Calculate the value of this expression.

$$2^{5} - (4^{2} - (12 - 7) \cdot \sqrt{9} + 1) + 18 : 6 + \sqrt{25}$$

Solution

Do the part in brackets first, following the order of operations.

$$2^{5} - (4^{2} - (12 - 7) \cdot \sqrt{9} + 1) + 18 : 6 + \sqrt{9}$$

brackets
$$= 4^{2} - 5 \cdot \sqrt{9} + 1$$

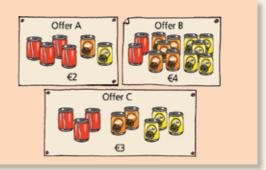
powers and roots
$$= 16 - 5 \cdot 3 + 1$$

multiplication and division
addition and subtraction
$$= 2$$

After the brackets, do the other operations, following the order again.

$$2^{5} - (4^{2} - (12 - 7) \cdot \sqrt{9} + 1) + 18 : 6 + \sqrt{25}$$
$$= 2^{5} - 2 + 18 : 6 + \sqrt{25} = 32 - 2 + 18 : 6 + 5$$
$$= 32 - 2 + 3 + 5 = 38$$

39 Calculate the value of these expressions. a) $19 - 3^2 + (\sqrt{49} \cdot (12 - 10) + \sqrt{4} \cdot (5 - 3))$ b) $\sqrt{36} : 2 + 4 \cdot (3 + 2^2 \cdot (9 - 6) + 1) - 3^2$ c) $5 + \sqrt{25} \cdot \sqrt{9} - (15 - 2^2 \cdot \sqrt{9})$





5 Calculations with powers

Multiplying and dividing numbers with the same base

• multiplying numbers with the same base

$$5^3 \cdot 5^4 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^{3+4} = 5^7$$
 Add the exponents together.

dividing numbers with the same base

$$5^{6}: 5^{4} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5} = 5^{6-4} = 5^{2}$$
 Subtract the exponents.

To multiply or divide numbers with the same base, keep the base as it is and add or subtract the exponents.

raising a power to a power

$$(5^2)^4 = 5^2 \cdot 5^2 \cdot 5^2 = 5^{2+2+2+2} = 5^{2-4} = 5^8$$
 — Multiply the exponents.

To raise a power to a power, keep the same base and multiply the exponents.

Numbers with an exponent of 1 or exponent of 0

$$2^{4}: 2^{3} = \begin{pmatrix} 2^{4-3} = 2^{1} \\ 2^{4}: 2^{3} = \\ 16: 8 = 2 \end{pmatrix} \xrightarrow{2^{1} = 2} 2^{4}: 2^{4} = \begin{pmatrix} 2^{4-4} = 2^{0} \\ 2^{4}: 2^{4} = \\ 16: 16 = 1 \end{pmatrix}$$

• A number with an exponent of 1 is equal to the base.

Any number with an exponent of 0 is equal to 1.

Numbers with the same exponent but different bases

• multiplying numbers with the same exponent but different bases

 $3^{4} \cdot 5^{4} = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (5 \cdot 5 \cdot 5 \cdot 5) = (3 \cdot 5) \cdot (3 \cdot 5) \cdot (3 \cdot 5) \cdot (3 \cdot 5) = (3 \cdot 5)^{4}$ $= 15^4$ — Multiply the bases together.

To multiply numbers with the same exponent but different bases, keep the exponent as it is, and multiply the base numbers.

dividing numbers with the same exponent but different bases

 $8^{3}: 4^{3} = (8 \cdot 8 \cdot 8): (4 \cdot 4 \cdot 4) = (8 : 4) \cdot (8 : 4) \cdot (8 : 4) = (8 : 4)^{3}$ $= 2^3 -$ Divide the bases.

To divide numbers with the same exponent but different bases, keep the exponent as it is, and divide the base numbers.

Take note

If a number doesn't have an exponent, it's considered to be to the power of 1. For example:

> $2 = 2^{1}$ $10 = 10^{1}$

 $23 = 23^{1}$

41 In your notebook, write each of these multiplications as a single power. a) $3^7 \cdot 3^3$ d) 9¹² · 9⁵ · 9³ b) $2^2 \cdot 2^5$ e) $5^3 \cdot 5^9 \cdot 5^6$ c) $8 \cdot 8^5$ **f)** $4 \cdot 4^8 \cdot 4$ 42 Copy and complete the missing exponents. a) $5^3 \cdot 5^{-1} = 5^7$ c) $7^7 \cdot 7 \cdot 7^{-1} = 7^{10}$ b) $2^{-1} \cdot 2^{6} = 2^{10}$ d) $8^{-1} \cdot 8^{3} \cdot 8 = 8^{11}$ 43 Simplify these to a single power. a) $3^7: 3^3$ c) $9^{12}: 9^5: 9^3$ b) $4^8: 4^2$ d) $5^{19}: 5^9: 5^6$ 44 Copy and complete the missing exponents. a) $5^8: 5^6 = 5^6$ c) $7^7: 7: 7^{-1} = 7^4$ **b)** $2^{-1}: 2^3 = 2^4$ d) 8^{-1} : 8^{3} : 8 = 845 Write each of these as a single power. a) $5^2 \cdot 5^3 : 5^4$ c) $7^{15}: 7^3 \cdot 7$ b) $2^6: 2^4 \cdot 2^3$ d) $9^{10}: 9^4 \cdot 9 \cdot 9^5$ 46 Write each of these as a single power. a) $(3^7)^2$ b) $(5^5)^3$ c) $(2^9)^5$ d) $(4^8)^2$ Opy and complete the missing exponents. a) $(3^{-1})^3 = 3^{12}$ c) $(2^{-1})^3 = 2^{30}$ b) $(5^4)^{=} = 5^{24}$ d) $(4^6)^{-1} = 4^{18}$ 48 Simplify these to a single power. a) $5^6 \cdot (5^3)^3 : 5$ c) $2^5: 2^3 \cdot (2^5)^3$ b) $(9^6)^2: 9 \cdot 9^2$ d) $3 \cdot (3^7)^2: 3^4$ 49 Write each expression using a single power. **b)** $4^6 \cdot 7^6$ c) $3^3 \cdot 4^3 \cdot 7^3$ a) $5^2 \cdot 3^2$ 50 Simplify these to a single power. a) $10^9: 5^9: 2^9$ **b)** $36^8: 2^8: 3^8$ c) $32^5:8^5:2^5$

51 Simplify these to a single power. a) $15^2 \cdot 3^2 : 5^2$ c) $24^4: 6^4 \cdot 3^4$ b) $18^5: 3^5 \cdot 6^5$ d) $6^7 \cdot 3^7 \cdot 2^7$ >>---- Worked example ----52 Write $4^4 \cdot (4^3 : 4)^3$: 4^6 as a single power. Solution Brackets $4^{4} \cdot (4^{3}:4)^{5} : 4^{6} \stackrel{\checkmark}{=} 4^{4} \cdot (4^{3-1})^{5} : 4^{6}$ Powers $= 4^4 \cdot (4^2)^5 : 4^6 = 4^4 \cdot 4^{2 \cdot 5} : 4^6$ From left to right $= 4^4 \cdot 4^{10} : 4^6 = 4^{4+10-6} = 4^8$ 4 53 Write each of these as a single power. a) $7^{6} \cdot (7^{3} : 7^{2})^{3}$ c) $(4^{7} : 4^{3})^{3} \cdot (4^{2})^{5}$ b) $(9^6 \cdot 9)^3 \cdot 9^2$ d) $(5^2)^3 \cdot (5^7 : 5^6)^8$ » Worked example — " " 54 Simplify this expression to a single power: $3^7 \cdot 2^7 : 6^2$ Solution $3^{7} \cdot 2^{7} : 6^{2} = (3 \cdot 2)^{7} : 6^{2} = 6^{7} : 6^{2} = 6^{7-2} = 6^{5}$ Numbers with the Numbers with the same base same exponent 55 Write each expression using a single power. a) $10^3: 5^3 \cdot 2^2$ c) $12^8:6^8:2^6$ **b)** $12^3: 2^3 \cdot 6^3$ d) $4^5 \cdot 2^5 \cdot 8^7$ 56 Write each expression using a single power. a) $5^3 \cdot 2^3$:10 **b)** $2^8: 2^3 \cdot 6^5$ **c)** $8^7: 8^3: 2^4$ 57 Write each expression using a single power. a) $16^2 \cdot (2^3)^4 \cdot 4^3$ b) $81^3 \cdot 3^7 \cdot 9^2$

(5) I Listen to the students simplifying to a single power. Write their answers and then decide if they're correct.
a) 7² · 7⁵
b) 5⁹ : 5³
c) 2⁷ · 3⁷
d) (6²)⁶
e) 18⁵ : 6⁵
f) (8³)⁵