OXFORD IB COURSE PREPARATION

PHYSICS

FOR IB DIPLOMA COURSE PREPARATION

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OXFORD

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Answers to questions in this book can be found at www.oxfordsecondary.com/9780198423591

Motion and force

The main purpose of science is simplicity and as we understand more things, everything is becoming simpler.

Edward Teller, Conversations on the Dark Secrets of Physics (1991)

Chapter context

This chapter deals with how things move and introduces you to some important related concepts that extend throughout physics—in particular, the **conservation of momentum**.

Learning objectives

In this chapter you will learn about:

- → distance and displacement, speed and velocity, and acceleration
- → displacement-time and velocity-time graphs
- → the kinematic (suvat) equations
- → forces and Newton's three laws of motion
- the effects of friction
- → work, energy and power
- conservation laws
- momentum and impulse.

1.1 Faster and faster

There are two aspects of motion to study—how it is defined and measured, and how it can be changed. This first section looks at the basic definitions and how they are linked.

DP ready Nature of science

Mass, length and time

All cultures measure the quantities used in everyday life, but they use many different units; for example, mass is measured in grams, ounces (to measure gold) and maunds (an Indian unit). Science uses a single agreed system of units that was established in 1960, though its development began well before that. It is called the SI (*Système Internationale d'unités*). Seven base units are defined as **fundamental** units. All other units are derived from these and are **secondary** units.

The fundamental SI units used in this book, together with their abbreviation and quantity, are:

- mass: the kilogram (kg)
- length: the metre (m)
- time: the second (s)
- electric current: the ampère (A)
- amount of substance: the mole (mol)
- temperature: the kelvin (K)

A fundamental unit not used in the Physics Diploma Programme is the candela (cd), the unit of luminous intensity.

Execute the second seco

- → displacement
- velocity
- acceleration
- → force
- Newton's laws of motion
- → work done
- power
- → momentum
- → impulse

DP link

In the IB Physics Diploma Programme you will learn about units when you study **1.1 Measurements in physics**.

You will learn about distance and displacement in **2.1 Motion**.

Distance and displacement

Imagine running round an athletic track. Figure 1 shows a map of the track. The total distance around the track on the inside lane is 400 m. When you have run halfway round, how far have you travelled?



Figure 1. Distance and displacement on a race track

One answer is 200 m—half the track length. This is the *distance* you have run. It is a measure of how much ground you have covered irrespective of direction. It is a *scalar* quantity.

But another way to look at it is that you are only 25 m away from your starting point and due north of where you began. This is your *displacement*, which is a *vector* quantity, and always requires a magnitude (the number part) and a direction (the start-to-finish information).

Continue running back to the starting point. Your distance travelled is now 400 m but your displacement has become zero.

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Question

- **a)** Calculate for your journey from home to school
 - i) your displacement (including direction)
 - ii) your distance travelled.
 - b) Identify how your answers to (a) change for your journey going from school to home.

Maths skills: Scalars and vectors

You need to know how to manipulate scalars and vectors. Here are the ground rules:

Adding and subtracting: Scalars are numbers, and are added and subtracted like ordinary numbers.

In adding or subtracting vectors you must take account of the direction as well as the size. The best way to see this is to begin with a scale drawing. Imagine that a boy cycles 3 km due north along a straight road and then 4 km along another road that goes due east (figure 2).

Key term

Distance is the length of a path travelled between two points. It is a scalar quantity.

Displacement is the

difference (in magnitude and direction) between an initial and final position. It is a vector quantity.

The units of distance and displacement are the metre (m).

A **scalar** quantity has only magnitude; a **vector** quantity has both magnitude and direction.

Internal link

Vectors and scalars occur in many topics, notably in the kinetic theory of gases (3.2 Gas laws).

DP link

In the IB Physics Diploma Programme, ideas about motion are studied in **2.1 Motion**, and vectors are studied in **1.3 Vectors and scalars**.



Figure 2

From figure 2, the total distance travelled is (3 + 4) = 7 km. What is the displacement? It is measured from the beginning of the journey, A, direct to the end of the journey, C. There are two ways to work this out.

One way is to use trigonometry. Compute the distance using Pythagoras's theorem: $\sqrt{3^2 + 4^2} = 5$ km

and compute the angle using $\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53^\circ$.

So the displacement is 5.0 km in a direction N 53°E.

The other way is by scale drawing. Draw the first vector upwards (north), 3 cm long (using the scale 1 km \equiv 1 cm). At the top end of this vector (which shows where the boy was after the first leg of the journey) draw a second line. This should be 4 × 1 cm, that is, 4 cm long, and should go to the right. Use a protractor (or squared paper) to ensure that the angle between the vectors is 90°. The displacement is the vector (called the resultant vector) that stretches from the start of the first vector to the end of the second vector. Measure it and it should be 5 cm long; use a protractor to check that the angle between the first vector and this resultant is 53°.

To subtract vectors by scale drawing, treat the vector being subtracted as though it had the opposite direction to its actual direction. Then add this new (negative) vector to the other.

The idea of adding a scalar to a vector in physics has no meaning. It is like adding an energy in joules to a velocity in metres per second.

Multiplying and dividing: Again, scalars are multiplied and divided just like ordinary numbers.

It is possible to multiply a vector by a scalar. The direction does not change, and the magnitude of the vector is multiplied by the scalar. A velocity of 10 m s⁻¹ in a direction due east that is multiplied by 5 becomes 50 m s⁻¹ still in the direction due east.

There are two ways to multiply vectors together (called "dot" and "cross" products); you may meet them in the IB Mathematics Diploma Programme, but they will not be required in Physics.

Table	1.	Examples	of	scalars
and v	ect	tors		

Examples of scalar quantities	Examples of vector quantities
mass	weight
speed	velocity
time	acceleration
energy	force
power	magnetic field strength
temperature	electric current

Internal link

There are some notes on trigonometry at the end of **1.2 Pushes and pulls**.



average speed

= total distance travelled total time taken

average velocity

= change in displacement time taken

The units of speed and velocity magnitude are metres per second (m s⁻¹; it is not usual to write m/s even though you might have done so in earlier study). Speed is a scalar quantity; velocity is a vector and always needs both magnitude and direction.

🜖 DP link

You will learn about significant figures and their treatment when you study **1.2 Uncertainties and errors**.

Table 2.

Time (s)	Distance from start (m)
0	0
1	2
3	16
5	44
7	86
9	142
11	205
13	275
15	345
17	415

Average speed and velocity

We often need to know not just the length of our journey but also how quickly we travelled. To do this we define two quantities that mirror the distance and displacement quantities: these are *speed* and *velocity*. For the calculation of speed or velocity we often use the time average. Time here is the travel time between measuring the first position and the second position.

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Worked example: Speed and velocity

1. Look again at the running track shown in figure 1. If it takes you 40 s to run halfway round, you cover the distance with an average speed of $\frac{200 \text{ m}}{40 \text{ s}} = 5.0 \text{ m s}^{-1}$. But the average velocity

is $\frac{25 \text{ m}}{40 \text{ s}} = 0.63 \text{ m s}^{-1}$ due north.

Maths skills: Significant figures, decimal places and standard form

The running track problem has its answers expressed to two significant figures (2 sf) because this was the smallest number of sf expressed in the data. Writing "200 m" implies that we know the value to the nearest metre, that is, 200 ± 1 m; this is 3 sf, whereas "40 s" implies that we know the time to the nearest second (2 sf).

Never quote an answer to better than the smallest number of sf in the data. And be careful with rounding when you adjust the final answer.

Decimal places (dp) are often confused with significant figures: 123.45 is a value quoted to 5 significant figures and 2 decimal places.

A good way to avoid being tripped up by sf and dp is to use standard form: 1.2345×10^3 . When you deal with small or very large numbers such as the mass of a proton (1.67×10^{-23} kg to 3 sf), standard form is crucial.

Question

- 2 A teacher walks 5 m north, 2 m east, 5 m south and 2 m west. The whole journey takes 42 s. Calculate the teacher'sa) average speed b) average velocity.
- 3 Give two examples of a **vector** quantity and two examples of a **scalar** quantity.

Instantaneous speed and velocity: distance-time graphs

Car drivers realize that an important fact about a car journey is not necessarily the average speed, but the speed that a roadside camera records! This is known as the **instantaneous speed**, the speed at one moment in time. For a car, it is the speed indicated by the speedometer. Graphs make it much easier to visualize speeds compared to data tables. To demonstrate, consider the data for the distance travelled by a car in a straight line during the first few seconds of a journey in table 2. These data could be laboriously transformed into a set of average speeds by working out the distance travelled between successive pairs and dividing by the time between them, but plotting the graph from the distance–time data shows details of the motion straight away. Figure 3 shows the data plotted as a graph of distance (*y*-axis) against time (*x*-axis) with the best-fit curve drawn.

The car moves slowly at the start, so the gradient of the graph is small. As time goes on the speed increases (the graph is steeper) until it becomes constant (a straight line beyond 10 s).

The instantaneous speed at a particular time can be determined from a distance–time graph by finding the gradient of the line at that time.





Figure 3. Distance–time data from table 2

If the graph is a curve, draw a tangent to the line at 7.0 s (if the graph is a straight line then this step is not needed). The tangent line should be as long as possible.

Read off the intercepts on the axes and work out the gradient from:

change in the *y*-direction

change in the *x*-direction .

The values for this example are on the graph.

Treating this as an equation, you will see that the

gradient = $\frac{\text{change in distance / m}}{\text{change in time / s}}$ = speed, measured in m s⁻¹ = 9.7, measured in m s⁻¹.

Always quote the quantity and the unit in the final answer for a gradient (so 9.7 m s^{-1}).

Ouestion The distance-time graphs below show the motion of three 4 objects, A, B and C. Α B С distance / m distance / m distance / m 25 15 10 0 0 ٢ Π N time / s time / s time / s

Describe the motion of the objects as fully as you can.

- A car travels at a steady speed of 22 m s⁻¹ for 20 minutes. Then the car goes a further distance of 35 km for 15 minutes. Calculate:
 - a) the distance travelled in the first 20 minutes
 - b) the average speed for the 35 minute journey.

Acceleration: speed—time graphs

The final quantity used to describe motion is *acceleration*. This is a measure of the rate at which velocity changes. The word "rate" is another way to say "change in [quantity] per unit time".

Table 3 gives data of the speed every second for a car that moves from rest. After one second the car goes from 0 to 1.5 m s^{-1} so the change in speed in the first second is 1.5 m s^{-1} .

In the time from 1 s to 2 s the change in speed is again 1.5 m s⁻¹ (= 3.0 – 1.5).

In the third second (2 s to 3 s), the speed change is (4.5 – 3.0), still 1.5 m s⁻¹.

So for this journey, in every second of the motion, the speed increases by 1.5 m s⁻¹. The change in speed is 1.5 m s⁻¹ per second; this is an acceleration of 1.5 (m s⁻¹) s⁻¹, written as 1.5 m s⁻².

Acceleration is a vector quantity (with direction), though you will not always know a direction. If in doubt as to what is needed, always assume it is a vector and quote a direction if possible.

Again, there is a distinction between average acceleration (the change in speed each second over a definite time interval) and the instantaneous acceleration (the change in speed each second at one instant in time). And again, a graph shows these distinctions (figure 4).

First, look at the overall shape of the graph and see what it shows: the object starts at rest (meaning it has zero speed at zero time). Then the speed increases steadily for the first 4.8 s. The gradient of this straight line (region OA) gives the acceleration. From 4.8 s to 8.0 s (region AB) the speed does not change; the gradient of the graph, and therefore the acceleration, is zero. From 8.0 s to 12.0 s the speed is decreasing, so the acceleration now has a negative value.

One term often used to describe a decrease in speed is "deceleration". Take some care with this: it is better to call the quantity "acceleration" and then to use a minus sign to make it clear that the gradient of the velocity-time graph (and therefore the acceleration) is negative.

Key term

acceleration

change in velocity time taken for change

The unit of acceleration

is m s⁻², which is the equivalent of $\frac{(m s^{-1})}{2}$.

Table 3.

Time (s)	Speed (m s ⁻¹)
0.0	0.0
1.0	1.5
2.0	3.0
3.0	4.5
4.0	6.0



Speed-time graphs and distance travelled

There is more information to be gained from a speed–time graph such as figure 4.

As discussed before, the quantities speed and time give acceleration speed

from $\frac{\text{speed}}{\text{time}}$. But notice that the definition for speed can be rearranged

to give distance = speed × time. The quantity (speed × time) represents the area under a speed-time graph. We can work out the distance travelled for part or all of a journey by calculating the area under the line for the speed-time graph. The units for speed × time are (m s⁻¹) × (s). The units of seconds cancel, leaving only metres, as you should expect.

Maths skills: Estimating the area under a speed-time graph



Figure 5. Calculating area by the use of regular shapes

- 1. Select the area for which you need to know the distance. Always calculate the area starting from the time axis (that is, from zero speed upwards). In the example in figure 5, this is particularly important for times between 8.0 s and 12.0 s.
- Divide the area into easily calculated regions, either rectangles or right-angled triangles. In figure 5, two triangles and two rectangles do the job. When there is a curved line you may have to estimate the area (figure 6).
- 5. Either count the squares in the grid (best for curves) or calculate the area (best for lines). Remember that the area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$ whereas a rectangle is base \times height.
- 4. Add together all the areas to get the total distance.

Area	Calculation	Distance / m
W	$\frac{1}{2} \times (4.8 - 0) \times 2.0$	4.8
Х	$(8.0 - 4.8) \times 2.0$	6.4
Y	$\frac{1}{2}$ × (12.0 - 8.0) × (2.0 - 1.0)	2.0
Z	(12.0-8.0)×1.0	4.0
Total		17.2

Table 4. Area calculation for the speed-time graph in figure 5

In the example the total distance travelled is 17.2 m— probably best expressed as 17 m to 2 sf.

If you cannot divide the area into regular shapes, then count the number of squares, as shown in figure 6.

There is a tick in every complete large square, and some ticks where incomplete squares are roughly equivalent to one large square, making about 19 squares altogether. An estimate to the nearest square is as good as you will be able to manage. Each square is 0.5 m s^{-1} by 2.0 s in area, in other words 1.0 m. So, 19 m underneath the graph in total.



Figure 6. Counting squares to estimate area



Figure 7. Velocity-time graph

Velocity-time graphs and displacement

A velocity–time graph can give even more information, this time including direction. Figure 7 shows a graph for a journey along a **straight line** (we need to know this, otherwise we cannot make some of the later deductions). As usual the gradients of the graph give the accelerations (also in the direction of motion). This time, however, the line goes below the *x*-axis. When it does so, the velocity is negative. This means that the object is now travelling back towards the starting point. The area is also negative and represents displacement back towards the starting position.

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Worked example: Analysing velocity—time graphs

2. Analyse as much as you can of the motion for figure 7.

Time / s	Analysis
0-10	Accelerating; acceleration is 0.40 m s ^{-2} ; displacement is 20 m in +ve (positive) direction
10-14	Slowing down to zero; acceleration is -1.0 m s^{-2} ; displacement is 8 m in +ve direction
14-18	Stationary; no change in displacement
18–23	Accelerating but towards starting point; $-1.4~m~s^{-2}$; displacement is $-17.5~m$
23–26	Slowing down so accelerating in +ve direction; +2.3 m s $^{-2}$; displacement is -10.5 m
0–26	Displacement is $+20 + 8 - 17.5 - 10.5 = 0$ m so object arrives back at starting point

Question

6 The graph shows the variation of speed with time for a car.



- a) State the maximum speed of the car.
- b) Calculate the acceleration for
 - i) the first 20 s of the motion
 - ii) the last 10 s of the motion.
- c) Determine the distance travelled in the first 40 s.
- d) Determine the average speed for the whole journey.

7 A series of speed–time graphs are shown for four different journeys, A, B, C and D.



- a) Compare the journeys of A and B in as much detail as you can.
- **b)** Describe journey C.
- c) Describe journey D.

Kinematic equations

Speed-time graphs are a good way to visualize motion, and to estimate acceleration and distance travelled. However, sometimes there is a better method for calculating either the speed or acceleration or distance travelled: the **kinematic equations**. These equations are sometimes called the *suvat* equations from the symbols used:

DP link

You will learn about the kinematic (*suvat*) equations in **2.1 Motion**.

- *s* **distance** travelled
- *u* initial **speed** *v* final **speed**
- a acceleration
- t time taken

To use these equations we assume that the **acceleration is constant** and does not change throughout the motion. The acceleration is said to be **uniform** when this is true.

DP ready Nature of science

The four kinematic equations make assumptions about the systems they describe. The most important is that the acceleration is constant. When this is not true, the equations are not valid (and you may be penalized in an examination for using them). An example is a skier moving down a hill with a varying slope. The acceleration down the slope will not be constant so the equations do not apply.

Another assumption is that we are dealing with point objects. We do not consider the mass or distribution of mass of the objects.

These equations apply to translation only, not rotation (though they can be extended to rotation, as you will learn if you study Option B of the IB Diploma Physics Programme).

The speed-time graph below (figure 8) shows the change in speed of an object over time *t*. Compare the starting and finishing speeds with the list of symbols above. The graph is a straight line and, of course, this tells us that the acceleration is constant. The derivations for the four equations, related to the graph, are shown below.



Figure 8. Deriving the suvat equations

Equation 1

The acceleration is the gradient of the graph, so acceleration, $a = \frac{\text{change in speed}}{\text{time for speed change}} = \frac{(v - u)}{(t - 0)}$ $a = \frac{v - u}{t}$ and therefore v = u + at.

Equation 2

The distance travelled (area under the graph) can be evaluated as two areas: area_{Δ} and area_{\Box}.

The total area, *s* (distance travelled), is

area_{$$\Delta$$} + area _{\Box} = $\frac{1}{2} \times (v - u) \times t + u \times t = \frac{1}{2} \times at \times t + u \times t$
(this also uses $v = u + at$ rearranged as $v - u = at$)
which becomes $s = ut + \frac{1}{2}at^2$.

Equation 3

Combine equations 1 and 2 and eliminate *t* to give $v^2 = u^2 + 2as$.

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Equation 4

```
Eliminate a from the others to give s = \left(\frac{v+u}{2}\right)t.
```

Worked example: Using the kinematic (suvat) equations

- 3. A car accelerates uniformly along a straight road, taking 13 s to change its speed from 8.0 m s⁻¹ to 34 m s⁻¹. Calculate
 - a) the acceleration of the car
 - **b)** the distance travelled by the car in the 13 s time period.

Solution

a) Begin by writing down what you know and what is required.

$$t = 13 \text{ s}$$

$$u = 8.0 \text{ m s}^{-1}$$

$$v = 34 \text{ m s}^{-1}$$

$$a = ?$$

so $a = \frac{v - u}{t} = \frac{(34 - 8)}{13} = 2.0 \text{ m s}^{-2}$
b) $s = ut + \frac{1}{2}at^2 = 8 \times 13 + \frac{1}{2} \times 2 \times 13^2 = 104 + 169 = 273 \text{ m}$

As all the data are to 2 sf, the distance travelled is best given as 270 m.

- **4.** An aircraft lands on a runway, taking 920 m to stop from a landing speed of 45 m s⁻¹. Calculate
 - a) the time to stop
 - **b)** the average deceleration.

Solution

a) s = 920 m $u = 45 \text{ m s}^{-1}$ v = 0 t = ?Equation 4 can be rearranged as $t = \frac{2s}{v+u} = \frac{2 \times 920}{0+45} = 40.9 \text{ s or } 41 \text{ s to } 2 \text{ sf.}$ b) One route is to use $v^2 = u^2 + 2as$

(it is important to link the values to the symbols: $45^2 = 0^2 + 2 \times a \times 920$ is wrong)

$$a = -\frac{2025}{2 \times 920} = -1.1 \text{ m s}^{-2} \text{ to } 2 \text{ sf}$$

So $0^2 = 45^2 + 2 \times a \times 920$

(notice the minus sign; it tells us that the aircraft is slowing down, so this is a deceleration)

Question

- 8 A motorcyclist accelerates uniformly from rest to a speed of 45 m s⁻¹ in 12 s. Then she brakes with a uniform deceleration to stop in a distance of 85 m.
 - a) Calculate, for the first 12 s of the journey,
 - i) the acceleration
 - ii) the distance travelled.
 - b) Calculate, for the second part of the journey,
 - i) the deceleration
 - ii) the time taken to stop.
 - c) Sketch a graph to show the variation of speed with time for this journey.
 - d) Use the graph to calculate the average speed for the whole journey.

Acceleration due to gravity

When an object falls from rest close to the Earth's surface, it accelerates downwards. The magnitude of this acceleration due to gravity, given the symbol *g*, can be measured by dropping a small ball from rest below an ultrasound sensor connected to a data logger.

DP link

You will learn about the acceleration due to gravity and its determination in 2.1 Motion and in 6.2 Newton's law of gravitation.



Table 5. Averaged results for the experiment in figure 9

Data loggers can usually be programmed to produce either a distance–time graph or a speed–time graph; the latter gives more information.

Table 5 gives the averaged speed-time results for three runs of this experiment. The results for 0.60 s and beyond show that the ball must have stopped moving somewhere between 0.55 s and 0.60 s. It probably hit the bench.

To find the value of *g*:

- 1. Begin by drawing the graph and then constructing the best-fit line (there is advice in the Maths skills section on page 12). Notice that there are some random errors in the measurements.
- **2.** Measure the gradient and use it to calculate *g*. Compare your answer with the accepted value.
- **3.** There is more you can find out from this graph. Think about the other quantity that a speed–time graph can give. What will it tell you in this experiment?



Figure 9. Measuring the acceleration due to gravity, *g*

Maths skills

Plotting graphs

- Use sensible scales for your axes: 1:1, 1:2, 1:5 are good; 1:3, 1:6, 1:7 and 1:9 are hard to use.
- A graph should occupy *at least* half the grid on the graph paper.
- To achieve the point above, consider using a false origin (one that is not (0,0)).
- Mark your data points consistently and clearly, use ×, +, \odot .
- All marks on the graph (plots or lines) should be drawn with a sharp pencil.
- Label axes correctly with the quantity / power of ten and unit, for example distance / 10^3 m.

Drawing a best-fit line

- Draw straight lines with a transparent ruler (so you can see all the points at once).
- Draw curves free-hand, in one movement that you have practised several times without putting the pencil to paper. Turn the paper before you start so that your hand is on the inside of the curve.
- Get a balance of points on each side of the line (whether straight or curved). Make the total distance from points to the line as small as possible.
- If there are error bars on the data, draw the line through all the error bars if possible.
- Don't force the line through the origin unless you are sure this is the correct physics for the situation.

Question

- 9 A cyclist accelerates uniformly from rest to a speed of 9.0 m s⁻¹ in a time of 45 s. Then he immediately applies the brakes and stops with uniform acceleration. The braking distance is 27 m.
 - a) Calculate, for the first 30 s,
 - i) the acceleration
 - b) Calculate, for the braking,
 - i) the acceleration

ii) the time taken to come to rest.

ii) the distance travelled.

- c) Determine the average speed for the whole journey.
- **10** Figure 10 shows the speed-time graph for a sprinter in a race.



Figure 10. Speed-time graph for a sprinter

Determine

a) the acceleration of the sprinter at the start of the race

(?

- **b**) the total distance travelled in 6.0 s
- c) the average speed of the sprinter over the first 4.0 s.

1.2 Pushes and pulls

You may have been taught that a *force* is a push or a pull that acts on something due to another object. In this section you will look at forces: what they are and what they do.

Balanced forces

Imagine a ball resting on a table on the Earth's surface (figure 11). The ball is not moving relative to the table or the Earth even though the gravitational pull of the Earth and other forces are acting on it. This is because all the forces are **balanced**. We say that the ball is in **equilibrium**.



Figure 11. Balanced forces between a ball and a table

However, a careful examination of the forces shows that the situation is more complex than this. As well as the gravitational effects, the surface of the table and the ball are deformed slightly by the gravitational forces that are acting. The diagram shows these four forces (the ball and the table are separated for clarity):

- the weight of the ball (the Earth's downward gravitational pull on it)
- the gravitational force of the ball on the Earth upwards (this has a tiny effect as the Earth is so large, but it exists)
- the spring force of the table upwards on the ball as the surface tries to return to being flat
- the spring force of the ball downwards on the table as the ball tries to return to being spherical.

The gravitational forces and the spring forces are balanced, so the net force (all the forces added together) is zero.

A crucial point to recognize here is that the directions of the forces are discussed as well as their magnitude. Forces are vectors and have both magnitude and direction.

DP link

You will learn about forces and Newton's laws of motion when you study **2.2 Forces**.

Key term

When a **force** acts on an object, the object moves if it is free to do so. More precisely, the force causes an acceleration. Unless, that is, some other force prevents the motion.

Internal link

Balanced forces are discussed further in the context of Newton's third law of motion later in this section, and frictional forces are covered in **Friction effects** at the end of this section.

Key term

Newton's first law of motion

states that, when no external force acts on an object, the object remains stationary or continues to move with a constant velocity.



Figure 12. A page from Newton's *Principia* in which he discusses the three laws of motion (and much more)

DP ready Nature of science

Galileo and Newton



Figure 13. Not how cannonballs really travel

Some more examples of cases in which forces are balanced include:

- an ice cube floating at rest in a glass of water
- an aircraft moving at a constant velocity, where the thrust and the air resistance are exactly in balance
- a child pulling a sled at a constant velocity along snow (the tension in the rope to the sled is equal to the friction at the snow surface).

Newton's first law of motion

If you live in a part of the world that is very cold in winter, you will be very familiar with the sled example above. When a sled is pushed on a horizontal surface it can travel for a long distance before stopping; the frictional force of the ice on the runners is small. What would happen if there were no friction at all? The answer is that the sled would continue to move at a constant velocity. This is the basis of *Newton's first law of motion*. A net force must act before an object's velocity can change (that velocity could be zero if the object is initially stationary). The use of velocity rather than speed here is crucial because, as you will see when you study circular motion, the direction of the force vector relates to the direction of the vector change in the velocity.

Newton was not the first to recognize the relationship between force and change in velocity. Galileo and others were beginning to come to this conclusion in Europe during the 16th century. Before then people thought that force had constantly to be supplied to enable an object to keep moving. This picture, drawn by Diego Ufano, a Spanish military engineer who died in 1613, shows how people once thought cannon balls moved in the air, running out of "force" just before they fall vertically to the ground (figure 13).



Galileo performed a "thought experiment" to help himself see what was happening. He imagined a ball,

released from rest, on a V-shaped ramp (figure 14). The ball reaches the same height on the righthand side of the ramp in a) and b) if no friction acts. What happens if the right-hand side of the V is made horizontal? Galileo realized that the ball would roll on for ever—no force, no change in velocity.

Newton recognized the importance of the work of earlier scientists to his own thinking. He said: "If I have seen further, it is by standing on the shoulders of giants"; he was probably using the idea of 12th-century philosopher Bernard of Chartres, who realized that truth almost always builds on previous discoveries.

Newton's second law of motion

Force, mass and acceleration are related by:

force = mass \times acceleration F = ma

- Notice that *m* is a scalar quantity and *a* is a vector; this is permitted, because the mass quantity simply multiplies the vector and does not change its direction.
- A consequence of this is that the direction of *a* and the direction of *F* are the same.
- Only one *F* is referred to in the equation. This is the **resultant force** or total force if there are two or more forces. You met the idea of adding vectors in *1.1 Faster and faster* in the "Scalars and vectors" section. You can use the drawing or calculation method for forces too.

Worked example: Newton's second law of motion

5. A car of mass 900 kg accelerates from rest to 15 m s⁻¹ in 50 s. Calculate the resultant force acting on the car.

Solution

Using v = u + at, $a = \frac{15 - 0}{50} = 0.30 \text{ m s}^{-2}$ So $F = 900 \times 0.30 = 270 \text{ N}$

Mass and weight

A gravitational force acts on any object in the gravitational field of another. This is usually only obvious to us when we are considering how the pull of gravity—the gravitational force due to the Earth—acts on us or any other object on the Earth: in other words, *weight*. In fact, all masses exert a gravitational force on each other. The larger the mass, the larger the force. The forces are very small unless we are dealing with something the size of a planet or a moon.

For a mass of 1 kg, $F = \text{mass} \times g = 1 \times 9.81 = 9.81$ N (roughly 10 N).

DP ready Theory of knowledge

Is mass constant?

Einstein, in his special theory of relativity, said mass was not a constant—and experimental evidence now backs up this theory. Einstein showed that observed mass increases when speed increases. If you study Option A of the IB Physics Diploma Programme, you will look at this phenomenon in detail. However, for most of the course, you can assume that mass is constant.

So, if weight is gravitational pull, what is mass? This is the amount of substance in an object. However, although this definition is correct, it is not very helpful. In fact, it is difficult to pin down the concept of mass exactly other than to say it is the quantity that responds to force by accelerating. You can regard mass as a constant that depends on the number of atoms in an object, whereas weight can vary over the Earth's surface (because the value of *g* varies over the surface) and if someone makes measurements on the Moon or a nearby planet.

🜔 Internal link

We will see another way to write the law in **1.4 Momentum and impulse**.

Key term

WE

Newton's second law of

motion states that F = mawhere *F* is the resultant force, *m* is the mass, and *a* is the acceleration.

One **newton** (1 N) is the force that will accelerate a mass of one kilogram (1 kg) by one metre per second per second (1 m s^{-2}) .

The newton has the fundamental units kg m s⁻². As always, it is important to use a consistent set of units in calculations.

🕨 Key term

Weight is the gravitational pull that the Earth exerts on an object.

Newton's second law of motion (Newton 2) helps here. Objects accelerate downwards near the surface in the Earth's gravity at 9.81 m s⁻². So Newton 2 indicates that the force *F* acting on the object must be equal to the mass *m* multiplied by *a*.

DP link

You will learn how the gravitational force depends on mass and distance between objects when you study **6.2 Newton's law** of gravitation and also **10.2 Fields at work** (higher level only).

DP ready Theory of knowledge

Inertial or gravitational mass?

It is possible to think of mass in two ways:

i) as the response of an object to the application of a force (in other words the smaller the acceleration from a standard force,

the larger the mass because $m \propto \frac{1}{a}$), or

ii) as the response of an object to the gravity field.

These two descriptions are not the same; they are both physically and philosophically different.

Physicists assume that 1 kg of gravitational mass is equivalent to 1 kg of inertial mass.

Practical skills: Verifying Newton's second law



To verify the equation F = ma it is necessary to carry out two experiments: one showing that $F \propto a$ while keeping *m* constant, and another showing that $a \propto \frac{1}{m}$ while keeping *F* constant.

The same apparatus can be used for both experiments.

$F \propto a$

- The force is applied to the trolley using an elastic thread held at a fixed extension. You can do this by keeping your hand in the same position relative to the trolley as the trolley accelerates.
- The measurement of the acceleration of the trolley can be made in a number of ways, including an ultrasound sensor connected to a computer (programmed to provide a direct readout of the acceleration), or a light gate that can time how long it takes the trolley to pass through (in this case you need to use the *suvat* equations to work out the acceleration).
- Measure the acceleration with one, two and three identical threads of the same initial length in parallel (one, two and three equal forces) all extended by the same amount.
- Plot a graph to show the variation of acceleration with force. You should find that the line is (approximately) straight and through the origin.
- Think carefully about errors and how to eliminate them. The friction at the axles of the trolley is a particular problem. What can you do to the table to eliminate this friction? What would you expect a trolley with no resultant friction acting on it to do (think Newton's first law)?

$$a \propto \frac{1}{m}$$

- This time you will keep the force the same for each run.
- Add mass to the trolley and measure the acceleration.
- This time plot a graph of *a* against $\frac{1}{m}$. It should be a straight line again.

Question

11 The acceleration due to gravity near the surface of Titan, a moon orbiting Saturn, is 1.3 m s⁻². A spacecraft is sent to Titan. It contains a payload with a mass of 250 kg.

- a) Calculate the weight of the payload on Earth.
- b) Calculate the weight of the payload in outer space.
- c) Calculate the mass of the payload on Titan.
- d) Calculate the weight of the payload on Titan.

Newton's third law of motion

The concept that underpins this law (which we will call Newton 3; the first law will be Newton 1 and so on) is not trivial, no matter how short the statement of the law. You may find it helpful to re-read the first section of this chapter before going on.

When A exerts a force on B, then B must exert an equal and opposite force on A. On the face of it, this is straightforward. The trick comes in correctly identifying the pair of forces, known as an **action–reaction pair**.

Go back to the ball sitting on the table from figure 11. There are two pairs of forces at work here:

- pair 1: the gravitational pull of the Earth on the ball, and the gravitational pull of the ball on the Earth
- pair 2: the force of ball on the table, and the force of the table on the ball—both arise from the deformation of the one by the presence of the other.

Notice that:

- within the pair, the magnitudes are the same, but the directions are opposite
- for this case where there is equilibrium, the magnitudes are the same for all four forces, but this will not be the case when there is acceleration.

Another example is a ball falling under gravity with no air resistance (figure 15).

The Earth exerts a pull on the ball (figure 15). The ball exerts a pull on the Earth of the same magnitude and opposite direction. This is an action–reaction pair (Newton 3).

The Earth's pull on the ball leads to the acceleration (Newton 2) that we identify as *g*. The pull of the ball on the Earth gives rise to an

acceleration of the Earth, but this is tiny because $a = \frac{F}{m}$ (Newton 2) and

m is very large. (You may also want to consider what happened to the Earth when the ball was originally moved into the air. Remember: the ball had to be accelerated using a force to move it above the surface.)

0

Key term

Newton's third law of motion states that every action has an equal and opposite reaction.

DP link

You will learn about action– reaction pairs when you study **2.2 Forces**.



Figure 15. An action and reaction pair

DP ready Nature of science

Estimating the acceleration of the Earth

We can estimate how small such effects on the Earth are. The mass of the Earth is about 6×10^{24} kg. What is the effect of a boy jumping down from a wall 1 m high?

As we only need a rough answer, call the mass of the Earth 5×10^{24} kg and assume the boy has a mass of 50 kg.

Because the force on the boy and the force on the Earth are the same, $m_{\rm Earth} \times a_{\rm Earth} = m_{\rm boy} \times a_{\rm boy}$ and therefore

 $\frac{a_{Earth}}{a_{boy}} = \frac{m_{boy}}{m_{Earth}} = \frac{5 \times 10}{5 \times 10^{24}} = 10^{-23}$. In other words, the acceleration of

the Earth due to the boy is only about 10^{-22} m s⁻².

The skill of making estimates will be an important one for you in both your theory classes and your practical work in DP Physics.

DP ready Theory of knowledge

Are Newton's laws really laws?

In science, the words "theory", "law" and "hypothesis" have a particular meaning that is more defined than in everyday language.

A theory is a model of some part of the universe. It can use facts, laws and hypotheses. A theory can be used to make a prediction that can be tested by experiment. Theories are often based on earlier theories.

Laws reflect observed patterns of behaviour and often take a mathematical form in physics. They usually do not attempt to explain an effect, but simply state what always happens.

A hypothesis is a possible explanation about the world that may or may not be true. Hypotheses can be tested by experiment and rejected if they prove incorrect.

Maths skills: Resolving vectors

To **resolve** a vector means to identify two vectors that add to give the original vector. For the IB Physics Diploma Programme this is limited to two vectors that are at 90° to each other. You will need to be able to do this to work out the effect of gravity on objects that are falling while also moving horizontally.

An example is the initial velocity of an object that is thrown into the air at an angle θ to the ground.



The vector can be resolved either by drawing or algebraically.

For the scale drawing method, draw the original vector and the two directions along which we want to resolve it (usually horizontal and vertical lines) beginning at the start of the vector. Then draw lines from the end of the vector parallel to these directions. These lines intersect with the direction lines at the ends of the two resolved components of the vector.

Algebraically, when the angle between the horizontal and the original vector is θ , then, by trigonometry, one vector is $v \cos \theta$ in the horizontal direction and the other is $v \sin \theta$ vertically.

Friction effects

In the real world, no surfaces are completely frictionless, the wind blows and air resistance acts. How do we take account of these in calculating motion?

At a simple level friction can be divided into two types: solid friction and fluid friction.

Solid friction is the friction observed when one solid surface is dragged over another. Take the case of a book being pulled across a table. Friction arises because the atoms in the book cover interact with the atoms in the table top. Change the materials and the amount of friction will change.

Fluid friction is the drag on objects when they move in gases (air resistance) or liquids (viscosity).

Not all friction is wasteful. We make good use of friction in many of the devices we use all the time. Imagine walking to school in a world with no friction!

When we consider friction forces, we encounter examples where there is more than one force acting on a moving object and the forces act in different directions. We need to consider how to deal with the addition of vectors when they are not at right angles to each other. This is best done using an example.

Worked example: Forces acting at angles

6. A girl is dragging a box across rough ground using a rope. The rope is angled upwards at an angle θ to the horizontal. What force does she need to exert so that the box moves horizontally at a constant velocity?



Solution

According to Newton 1, because there is no change in velocity there must be no resultant force. Therefore in the horizontal direction the force to the left must be equal to the horizontal component of the tension in the rope, and in the vertical direction the weight downwards must be equal to the vertical component of the tension in the rope. Resolving the forces horizontally shows that $F = T \cos\theta$. Resolving the forces vertically shows that $mg = T \sin\theta$.

We can take the solution in Worked example 6 one step further and divide these equations to show that $\frac{T\sin\theta}{T\cos\theta} = \frac{mg}{F} = \tan\theta$.

DP link

You will learn about friction and treating it quantitatively when you study **2.2 Forces**. You may also meet fluid friction if you study **Option B.3 Fluids and fluid dynamics**.



Internal link

You will find a description of the phases of matter in **3.1 States of matter**.

Maths skills: Trigonometry basics

Sine, cosine and tangent



When γ is 90° (a right angle), $\sin \beta = \frac{b}{c}$, $\cos \beta = \frac{a}{c}$, $\tan \beta = \frac{b}{a}$. Pythagoras's theorem

When γ is 90°, $c^2 = a^2 + b^2$

Sine and cosine rules

For any values of angles α , β , γ :

 $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$ (the sine rule) and $c^2 = a^2 + b^2 - 2ab\cos\gamma$ (the cosine rule).

Question

- 12 A block of mass 4.5 kg slides down a ramp at an angle of 30° with a constant acceleration. It travels a distance of 2.5 m from rest in 5.0 s.
 - a) Calculate the acceleration of the block.
 - b) Calculate the frictional force that opposes the motion of the block.



Internal link

You will gain an insight into how humans use elements of the natural world to generate electrical energy in **6 Generating and using energy**.

DP link

You will learn about energy and work done when you study 2.3 Work, energy and power.

1.3 Work and energy

In this section we examine the concepts of energy, power and efficiency. They form the backbone of physics and engineering because it is the transfer of energy from one place to another that allows us to extract useful work. The ability of humans to develop tools to transfer energy from one form to another has made us an adaptable and resourceful species.

Work done

Physicists have a precise, clear meaning for work: work is done when a force leads to the movement of an object. This leads to a definition of *work done* and a unit for energy, the *joule*.

Worked example: Calculating the work done

7. A force of 15 N acts on a mass and moves it against friction through a distance of 25 m in the direction of the force. Find the work done.

Solution

 $W = F \times s = 15$ (N) $\times 25$ (m) = 375 J.

8. A railway truck moves on rails that are laid west to east. A force of 1.5 kN acts on the truck and it moves 52 m.



Calculate the work done if the force acts:

- a) along the direction of travel of the track
- **b)** at an angle of 60° to the track.

Solution

- a) Work done = $F \times s = 1500$ (N) $\times 52$ (m) = 7.8×10^4 J or 78 kJ
- **b)** The component of force in the direction of motion is $F \cos 60$, which is $1500 \times \cos 60 = 750$ N.

So the work done is $750 \times 52 = 39\ 000\ J\ (39\ kJ)$

Maths skills: Using standard form in answers to questions

It is best to write the answer to part (a) of Worked example 8 in the form 75 kJ or 7.5×10^4 J because using 75 000 J could be taken to mean that you know your answer accurately to 5 significant figures. As the data in the calculation were to 2 sf it is wrong to imply this 5 sf level of accuracy.

Question

- **13** Calculate the work done when a force of 12 N moves an object through a distance of
 - a) 6.0 m
 - b) 6.0 m at 45° to the direction of the force.

Energy stores and pathways

Where does the energy to do work come from? Physicists use the concept of the "energy store". Energy is available for use as work when it moves from one store to another. Sometimes the origin of the energy is called the "source" and the store to which the energy goes is called the "sink". Here are some examples.

When the north poles of two magnets face each other, as you try to push the magnets together you store energy in the magnet system. Release the magnets and they fly apart. This repulsion could be harnessed to obtain work (two magnets, one on a railway truck and one on a fixed part of the track, could move the truck).



Key term

Work done = force \times distance moved in the direction of the force

$W = F \times s$

The unit of work is the **joule** (abbreviated J). One joule of work (1 J) is done when a force of 1 newton moves an object through 1 metre in the direction of the force.

The joule is not one of the fundamental SI units. From the equation, you can see that it can also be written as newton metres (N m), and this can be further written as kg m s⁻² \times m, in other words kg m² s⁻².

Notice that energy is a scalar quantity; it does not have a direction associated with it.



Figure 16. Energy transfer with a camping stove

😥 Internal link

Many of these energy stores are important for the generation and storage of energy:

- electrostatic and magnetic – 2 Electric charge at work
- nuclear 6 Generating and using energy
- thermal 3 Thermal physics
- global resources –
 6 Generating and using energy.

Numerical ways to represent energy stores and pathways are explored in **6.2 Energy resources**, where there is a discussion of Sankey diagrams.

Key term

The **principle of conservation of energy** states that energy cannot be created or destroyed. A spring or a rubber band has no ability to do work when unstretched, but stretch it in your fingers and it can release its store of energy as a catapult.

A camping stove contains propane fuel. When this gas mixes with oxygen from the air, the chemical bonds in the two gases are altered, releasing energy in the stove flame (figure 16). Often people refer to the propane as the store, but it cannot release energy unless oxygen is present, so it may be more correct to regard both gases as the store. There are many types of energy stores:

- chemical (for example, the propane stove described above)
- elastic (for example, the spring described above)
- electrostatic (energy stored in a system of two electric charges that attract or repel when released)
- gravitational (energy stored in a system of two masses that are attracted by gravity where one can be allowed to move relative to the other)
- kinetic (energy stored in a moving object)
- magnetic (for example, the magnets described before)
- nuclear (energy stored in atomic nuclei, transferred by radioactive decay, nuclear fission or nuclear fusion)
- thermal (energy stored in a hot object).

The principal energy pathways are:

- electrical (a charge moving through a potential difference)
- heating (when there is a difference of temperature)
- mechanical (a force moving an object through a distance)
- radiation (typical wave motion, eg light/radio/sound waves).

Conservation of energy

When energy transfers from one store to another, observations suggest that none is lost provided we are very careful to include every possible form of energy in our measurements. This is known as the *principle of conservation of energy*.

DP ready Nature of science

Conservation laws

Some physical quantities are always conserved. These laws are of great importance in the philosophy of the subject as well as in calculations. The laws include:

- conservation of charge
- conservation of linear and angular momentum
- conservation of energy (with the proviso that all energy forms must be considered).

Similarly, there are some fundamental constants that are thought to never change (the charge on the electron is an example).

Physics underwent a **paradigm shift** at the beginning of the twentieth century when Einstein suggested that mass was itself a form of energy, in his famous equation $\Delta E = c^2 \Delta m$. Δ stands for "change in" and *c* is the speed of light in a vacuum. This equivalence means that the principles of conservation of mass and conservation of energy can be combined.

DP ready Theory of knowledge

Paradigm shifts

From time to time in science a discovery or suggestion is made that is so different from the previous view that it causes a major shift in our models of the universe. Examples include:

- Galileo Galilei's observation of Jupiter's moons, which provoked his support for the Copernican view of the solar system
- Isaac Newton's work on the implications for the gravitational force
- Albert Einstein's recognition that time was not an absolute quantity but depends on the conditions of the observer.



Figure 17. Galileo Galilei (1564–1642), Isaac Newton (1642–1726) and Albert Einstein (1879–1955)

These are topics you are likely to discuss not only in your Diploma physics lessons but also in theory of knowledge.

Worked example: Drawing energy stores and energy pathways

9. An electric cell is connected to a motor that is raising a load. Draw the energy stores and the energy pathways for this process.

Solution



Kinetic energy and work

In a petrol-driven car, the chemical store of energy consists of the liquid fuel + oxygen in the air. It supplies energy that is eventually transferred into thermal stores, including raised temperature of the friction brakes when the car stops, heated air from air resistance, hot tyres from friction, and so on. Such stores probably cannot be used again, so this energy is "lost" to us. However, while the car is moving the energy that it has is called its *kinetic energy*.

The kinetic energy of an object increases when:

- the speed of the object increases for a given mass
- the mass of the object increases for a given speed.

Internal link

Energy pathways in a resource context are discussed in **6.2 Energy resources**.

Suppose that the car and passengers with a total mass m change speed from an initial speed u to a final speed v. This acceleration a takes a time t and occurs in a distance s.



So the work done *W* is $W = \text{force} \times s = m \times a \times s$ (*F* = *ma* has been used here to substitute for *F*).

However, *s* can also be replaced using the second kinematic equation $(v^2 = u^2 + 2as)$ to give

$$W = m \times a \times \left(\frac{v^2 - u^2}{2a}\right)$$

Cancelling the *a* and rearranging gives

$$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$$

The symbol used for kinetic energy in DP Physics is E_k .

The kinetic energy of an object of mass m moving at a speed v is

 $\frac{1}{2}$ × mass of the object × its speed².

This confirms the two predictions of how kinetic energy will vary with speed and mass listed on page 23.

Worked example: Calculating change in kinetic energy

10. A bus of mass 10 000 kg accelerates from a speed of 10 m s⁻¹ to a speed of 15 m s⁻¹. Calculate the change in the kinetic energy of the bus.

WE

Solution

The change in kinetic energy is $\frac{1}{2} \times m \times (v^2 - u^2) = \frac{1}{2} \times 10^4 \times (15^2 - 10^2) = \frac{1}{2} \times 10^4 \times (225 - 100) = 6.3 \times 10^5 \text{ J}$

Notice that $(15^2 - 10^2)$ is **not** the same as $(15 - 10)^2$. This is a common error: one is 125, the other is 25!

Question

- **14** Calculate the kinetic energy of:
 - a) a tennis ball of mass 58 g moving with a speed of 65 m $s^{\mbox{--}1}$
 - b) a boy on his bicycle of total mass 75 kg moving with a speed of 8.5 m s⁻¹.
- **15** Estimate the kinetic energy of:
 - a) a girl jumping off a wall of waist height when she reaches the ground
 - b) a car being driven at the speed limit
 - c) a bird in normal flight.

DP ready to learning

Units of kinetic energy

It is always sensible to check that the units of a new quantity (in this case, $\frac{1}{2}mv^2$) have the correct

fundamental units.

Ignoring the factor $\frac{1}{2}$,

which has no units, mv^2 is

kg × (m s⁻¹)², which expands to kg × m² × s⁻².

This rearranges to

 $(kg \times m \times s^{-2}) \times m$, which is N × m. In other words, it is force × distance; this was our original definition of work done.

DP ready Approaches to learning

Making estimates

One of the command terms used in IB Physics Diploma Programme questions is "estimate". When you are asked to estimate, you may be expected to "guess" some, or all, of the quantities involved in the problem, as in the estimate of the acceleration of the Earth in *1.2 Pushes and pulls*. The point of an estimate is to give a power-of-ten value for the answer, not an exact answer.

Outside examinations, use the internet to research reasonable values of quantities you do not know.

It is sensible to use estimates for physical constants. Use 10 m s⁻² for g rather than 9.81 m s⁻² if you just need a power-of-ten answer.

Gravitational potential energy and work

The term "potential" in physics requires care. It also occurs in the term "potential difference" in electrical theory (often abbreviated to pd). Some books emphasize that elastic energy (in a compressed spring, for example) is elastic **potential** energy in the sense that the spring has not yet been released to make its energy available.

Gravitational potential energy (gpe, with the symbol E_p) is energy stored in a system formed by the gravitational interaction of two (or more) objects. All massive objects (meaning objects with mass, not necessarily very big ones) attract each other by gravity and will therefore move towards each other if they can. (Gravitational repulsion is never observed.) When the two objects are held apart, they have the *potential* to move back together, and the energy released from the system can be transferred into useful work. A good example is a hydroelectric power station: water is allowed to flow downwards through a turbine. The turbine gains kinetic energy (it rotates), transferred to it from the kinetic energy of the water that in turn comes from the water's store of gravitational potential energy.

In this and other examples of transfer of gravitational potential energy (gpe), you should carefully analyse the sources and endpoints of energy and the transfer pathways for the energy. As with kinetic energy, your thinking about gpe should be quantitative as well as qualitative. We begin with the definition of work done and extend it to the case of a mass just above the Earth's surface (figure 18).

The mass is a distance h above the Earth's surface. Suppose that the acceleration due to gravity over this distance is effectively constant (it decreases as we move away from the surface but we can ignore this for small height changes). The work done is force × distance, as usual, so in this case work done = weight × height, which is $mg \times h$

so change in gpe = mass \times acceleration due to gravity \times vertical distance change.

When the object moves away from the Earth's surface the change in gpe is positive; when the movement is towards the surface the change is negative and the object loses gpe.

DP link

You will study kinetic energy and gravitational potential energy in **2.3 Work, energy and power**.



Figure 18. Gravitational potential energy of a mass above the Earth's surface

)P ready to learning

Units of gravitational potential energy

Gravitational potential energy is weight \times height, so this is a force \times distance expression and so is equivalent to work done in joules.



Changes in kinetic energy and gravitational potential energy

 $\Delta E_{\rm k} = \frac{1}{2} m \left(v^2 - u^2 \right)$

 $\Delta E_{p} = mg\Delta h$

where *m* is mass, *v* and *u* are final and initial speed, *g* is the acceleration due to gravity and Δh is the change in vertical height. Remember that Δ means "change in".

🜖 DP link

Later in the IB Physics Diploma Programme (10.1 Describing fields) you will learn to distinguish between "gravitational potential" and "gravitational potential energy". The former is the energy stored in the two-object system *per unit mass* and is quite different from gpe.

🕨 Key term

energy transferred

time taken for the transfer

The unit of power is the **watt** (symbol W). 1 watt \equiv 1 joule per second (1 J s⁻¹).

1 W is therefore also 1 N m s⁻¹, which in fundamental (SI) units is kg m s⁻² × m × s⁻¹ or kg m² s⁻³.

The watt is a small unit so you should become familiar with expressing power in kW (in domestic situations) and MW or GW (for energy generation on a local or national scale).

Worked example: Kinetic and gravitational potential energy calculations

11. A stone of mass 0.50 kg is dropped from rest from a point 2.5 m above the ground. Air resistance is negligible. Calculate the speed of the stone when it hits the ground. Take g = 9.81 m s⁻².

Solution

This problem can be solved in two ways: either by using energy ideas or by using the *suvat* equations from *1.1 Faster and faster*. The equations can be used here because the acceleration is uniform. The solution shown uses energy ideas.

The loss of gpe when the stone falls to the ground is $mg\Delta h = 0.50 \times 9.81 \times 2.5 = 12.26 \text{ J}$

This *loss* of gpe must equal the *gain* in kinetic energy so

$$\frac{1}{2}mv^2 = 12.26 \text{ J. So } v = \sqrt{\frac{12.26 \times 2}{0.50}} = 7.0 \text{ m s}^{-1}.$$

(Notice that you do not even need the value for mass here. Equating the two energies gives $\frac{1}{2}mv^2 = mgh$ so the mass *m* cancels to give $\frac{1}{2}v^2 = gh$ and therefore $v = \sqrt{2gh}$.)

Question

- **16** A stone has a mass of 3.5 kg. Assume g = 9.81 m s⁻². Calculate for the stone:
 - a) the gravitational potential energy as it is released from rest from a cliff 7.8 m above the sea surface

0

- b) the kinetic energy when it has fallen vertically through 3.6 m
- c) the kinetic energy just before it hits the sea.

Power

Two cyclists, each with the same total mass of 95 kg, ride up the same hill. The hill is 55 m high. This means that they both transfer 51 kJ ($mgh = 95 \times 9.81 \times 55$) to their store of gravitational potential energy. Cyclist A takes five minutes to climb the hill and cyclist B takes three minutes longer. A is transferring energy into the gravitational form faster than B; we say that A is more powerful than B. *Power* is the rate at which energy is transferred. In this case, cyclist A transfers energy to the gravitational field at a *rate of* $\frac{51000}{5 \times 60} = 170$ W;

B does so at 106 W $\left(\frac{51000}{8 \times 60}\right)$.

Power can be expressed in another way.

The expression for work done is force × distance. So power must be $\frac{\text{force} \times \text{distance}}{\text{time}}$. But this is also force × $\frac{\text{distance}}{\text{time}}$, in other words

force × speed. In symbols, power *P* can be expressed as $P = F \times v$.

Worked example: Calculating power

12. A lift door of width 1.2 m requires a force of 250 N to open it. An electric motor opens the door in a time of 6.0 s. Calculate the power that the motor must deliver.

Solution

The speed at which the door opens is $\frac{1.2}{6.0} = 0.20 \text{ m s}^{-1}$. The power delivered by the motor must be $250 \times 0.20 = 50 \text{ W}$.

Efficiency

The two cyclists described before transferred energy at different rates, but that was not the whole story. We calculated only the rate at which energy transferred into the gravitational potential energy store. There are many other ways in which energy transferred by the rider is lost: to air resistance, to rolling resistance of the tyres, to the chain connecting the pedals to the rear wheel, in the gear train, and in the muscles of the rider. All these factors reduce the *efficiency* of the system. Efficiency is the ratio of the useful work done by a system to the total energy transferred in all forms.

Worked example: Calculating efficiency

- **13.** An athlete on an exercise bicycle pedals against a resistance force in the bicycle of 25 N. Her speed on the bicycle is equivalent to a ground speed of 12 m s⁻¹.
 - **a)** Calculate the rate of transfer of energy to the bicycle by the athlete.
 - **b)** The athlete's muscles have an efficiency of 20%. Calculate the rate at which energy is supplied to her muscles by her body.

Solution

- **a)** $P = Fv = 25 \times 12 = 300 \text{ W} (3.0 \times 10^2 \text{ W} \text{ is better, to get the significant figures to match).}$
- **b)** She outputs 0.3 kW of energy each second. This is 20% $\frac{1}{5}$ of

the chemical energy required from her body, so her body supplies $0.3 \times 5 = 1.5$ kW (or 1.5 kJ every second) to her muscles.

Maths skills: Powers of ten

- Powers of ten are an effective way to avoid using large numbers of zeroes: 3 × 10⁸ m s⁻¹ is a better way to express the speed of light than 300 000 000 m s⁻¹.
- Powers of ten allow you to manipulate large and small numbers with ease.

A thermal power station produces 2×10^8 W of electrical power. When 1 kg of fuel is burnt it produces 8×10^7 J of energy. The efficiency of the station is 20%.

Calculate the mass of fuel burnt in one second.

The station requires $2 \times 10^8 \times \frac{100}{20} = 10^9$ J of energy to be supplied each second.

🜖 🛛 DP link

You will learn about power and efficiency when you study **2.3 Work, energy and power**.

Key term

You will often see the phrase **rate of** ... used. "Rate of transfer of energy" is an example in Worked example 13. This means the energy supplied per second.

So power is the rate of transfer of energy because it is the energy transferred in one second.

Key term

Efficiency can be defined in terms of either the energy transferred or the power input and output:

useful work done

efficiency = total energy transferred useful power output

or total power input Efficiency has no units because it is a ratio of two identical quantities. Efficiency can be expressed as a fraction, as a decimal number between 0 and 1, or as a percentage.

WE

Maths skills: Rules for manipulating powers of ten (indices) $10^{0} = 1$ $10^{m} \times 10^{n} = 10^{m+n}$ $10^{m} \div 10^{n} = 10^{m-n}$ $(10^{m})^{n} = 10^{m \times n}$

The mass of fuel – energy required each second	$-\frac{1 \times 10^9}{-13 \text{ kg}}$
energy from one kilogram	=1000000000000000000000000000000000000
in each second.	

• Science has a list of **prefixes** that can be added in front of units to avoid even the powers of ten. The full list is:

Prefix	Unit	Value	
peta	Р	10 ¹⁵	
tera	Т	1012	
giga	G	10 ⁹	billion
mega	М	10 ⁶	million
kilo	k	10 ³	thousand
hecto	h	10²	hundred
deca	da	101	ten
deci	d	10 ⁻¹	tenth
centi	С	10-2	hundredth
milli	m	10-3	thousandth
micro	μ	10 ⁻⁶	millionth
nano	n	10 ⁻⁹	billionth
pico	р	10-12	
femto	f	10-15	

Italics represent prefixes in the SI system that are seldom used.

Question

17 A 75 W electric motor raises a mass of 2.5 kg through a height of 1.8 m in 8.0 s.

Calculate:

- a) the electrical energy supplied to the motor
- b) the gravitational potential energy gained by the load
- c) the efficiency of the motor.

1.4 Momentum and impulse

Up until now, Newton 2 has been expressed as F = ma. This can be rewritten in terms of change in velocity:

 $F = m \times \frac{\text{change in velocity}}{\text{time taken for change}} = m \times \frac{\Delta v}{\Delta t}.$

 Δ stands for "change in …". So Δv is the change in velocity and Δt is the change in time. Another rearrangement of the same equation yields: $F \times \Delta t = m \times \Delta v$ and something interesting appears. Think about the case of an object falling to the Earth. Because there is an action–reaction pair, Newton 3 tells us that the forces acting on the object and the Earth are the same in magnitude (*F*). The two forces must obviously act for the same time (Δt) and therefore everything on the left-hand side of the equation is the same for both bodies. This means that the quantity (mass × change in velocity) must also be the same for

DP link

You will learn about momentum and impulse when you study **2.4 Momentum and impulse**. both bodies. This new quantity is of such importance in science that it is given its own name: *impulse*.

It leads us to another new, important quantity: *momentum*. Momentum is the product of (mass \times instantaneous velocity) for an object.

Conservation of momentum

The total momentum of a system remains constant providing no external forces act on it.

This law comes in two parts, and it is important not to overlook the second part, "providing no external forces act", because when a force outside a system acts on it, then the momentum can, and usually does, change.

When two railway trucks collide then at the moment of collision the momentum is unchanged, but as friction in the form of air resistance begins to act on both trucks, then the momentum of the system gradually decreases as energy is transferred into the air.

Momentum is one of the quantities in science that is **always** conserved. The total momentum is never lost, whatever the nature of the interaction. If you think some may have disappeared, you are not looking carefully enough at the problem.

Worked example: Conservation of momentum in collisions

14. Figure 19 shows two trolleys approaching each other. After colliding, they stick together.

Calculate the final velocity of the combined trolleys immediately after the collision.





Solution

The initial momentum (treating movement to the right as positive) = $m_1u_1 = m_2u_2 = 2 \times (+2) + 3 \times (-2) = 4 - 6 = -2 \text{ kg m s}^{-1}$

(this means the total momentum is 2 kg m $s^{\mbox{--}1}$ to the left).

After the collision, the momentum must be unchanged, but now the mass is 2 + 3 = 5 kg.

The final velocity = $\frac{\text{momentum}}{\text{total mass}} = \frac{-2}{5} = -0.4 \text{ m s}^{-1}$. This is negative so the joined trucks are travelling at a speed of 0.4 m s⁻¹ to the left.

🚽 Key term

Momentum = mass × velocity

The units of momentum are kg m s^{-1} .

The symbol p is often used for momentum and Δp for momentum change.

Momentum is a vector quantity; always specify its magnitude and direction.

Impulse = force × time for which it acts

Impulse = change in momentum



WE

You meet conservation laws a number of times in this book, including conservation of charge (**2.1 Electric fields and currents**) and conservation of momentum (**3.2 Gas laws**, discussing the motion of gas particles).

Question

- A 12 kg mass travelling to the left at 7.5 m s⁻¹ collides with a 3.0 kg mass travelling to the right at 5.0 m s⁻¹. The masses stick together when they collide and continue along the original line.
 - a) Calculate the initial momentum of the 12 kg mass and the 3 kg mass.
 - b) Calculate the initial total momentum.
 - c) Calculate the final velocity of both masses.

Energy changes in collisions

Look again at the example of the two trolleys colliding in Worked example 14 on page 29. Before the collision the two trolleys had a combined kinetic energy of $\frac{1}{2} \times 2 \times 2^2 + \frac{1}{2} \times 3 \times 2^2 = 4 + 6 = 10$ J. Remember that energy is a scalar and so there is no reason to consider the direction of the trucks.

After the collision the combined mass of 5 kg had a speed of 0.4 m s⁻¹. So the final kinetic energy was 0.40 J.

When energy is lost in a collision it is known as an *inelastic* collision. When energy is neither gained nor lost the collision is known as *elastic*. Where energy is input to the system during the collision, it is called a *superelastic* collision or explosion.

For the colliding trolleys, although the momentum of the system was conserved, the kinetic energy was not. Where has it gone? The answer is that the kinetic energy has transferred into several energy sinks. When the trucks collided, energy was used to operate the coupling that joins them together, sound energy was transferred to the air and parts of the truck mechanisms may have deformed, permanently or temporarily. All these changes require an energy transfer. Most or all of this energy will eventually find its way into a thermal (heat) form and become lost to us. For example, the energy transferred into sound waves will be dissipated in the air or nearby objects, leading to very small increases in temperature, which we cannot access to do useful work.

Worked example: Conservation of kinetic energy in collisions

15. A stone of mass 5.0 kg slides across ice on a pond and collides elastically at a speed of 0.20 m s^{-1} with a stone also of mass 5.0 kg. Calculate the final speeds of both stones after the collision.

Solution

Initial momentum of the stone is $p = mv = 5.0 \times 0.20 = 1.0$ kg m s⁻¹ Initial kinetic energy of system $E_k = \frac{1}{2} mv^2 = \frac{1}{2} \times 5.0 \times 0.20^2 = 0.10$ J

If after the collision the original stone has mass m_1 and speed v_1 , there are two equations, one for final momentum and one for final kinetic energy:

 $1.0 = m_1 v_1 + m_2 v_2$ $0.1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

The solution of these equations is that either $v_1 = 0$ and $v_2 = 0.2$ m s⁻¹ or that $v_2 = 0$ and $v_1 = 0.2$ m s⁻¹. The second solution can be discounted (it is as though there were no interaction). The first solution says that the final speed of the second stone is the same as the original speed of the first stone. The first stone stops completely, and the second stone moves off in the same direction at the original speed. This is an elastic collision as required.

DP link

You will learn about energy changes in collisions in 2.4 Momentum and impulse.

Key term

In an **elastic collision**: momentum is conserved and energy is conserved.

In an **inelastic collision**: momentum is conserved but energy is transferred out of the system.

In a **superelastic collision**: momentum is conserved but energy is transferred into the system.

Question

- 19 Railway truck A has a mass of 800 kg and railway truck B has a mass of 1600 kg. Truck A travels towards truck B at a speed of 8.0 m s⁻¹. Truck B is initially stationary. The trucks join during the collision.
 - a) Calculate the speed of the trucks immediately after the collision.
 - b) Calculate the total kinetic energy lost during the collision.
- **20** A hammer of mass 5.0 kg hits a thin vertical piece of wood that has a mass of 0.75 kg. The speed of the hammer is 2.5 m s⁻¹ when it hits the wood, and it does not bounce. The wood is driven 7.5 cm into the ground.
 - a) Calculate the vertical speed of the wood immediately after the hammer hits it.
 - b) Determine the average frictional force acting on the wood due to the ground.

Motion in a circle

Many objects move in a circular path. Obvious examples are the path of the Moon or a satellite around the Earth and of the Earth around the Sun. (Although these orbits are not perfectly circular, they are close enough for our purposes.) Likewise, the path of an object being whirled around in a horizontal circle springs to mind.



Figure 20. Centripetal force for an object on a string

Figure 20 shows the situation for an object rotating with a constant speed at the end of a string. What keeps the object rotating? The horizontal component of the tension in the string is providing a force inwards towards the centre of the circle. This force, which is keeping the object in its orbit, is the **centripetal force**.

Looking at all the forces acting on the object leads to figure 21.



DP link

You will study quantitative details about centripetal motion when you study **6.1 Circular motion**.

Figure 21. The forces acting on the rotating object

The centripetal force is provided by the horizontal component $T \sin \theta$, while the vertical component of the string tension balances the weight force, so $T \cos \theta = mg$. This explains why the string will never be completely

horizontal, as some vertical component of *T* always has to compensate for the weight of the rotating object. What determines the size of the component of the force that acts to keep the object rotating in this way? The centripetal force is increased when:

- the speed of the object or the mass of the object is increased
- the radius of the circle is reduced.

Circular motion is an example of a vector velocity that is **not** constant. The object has a constant speed, but the velocity is always changing (figure 22).



Figure 22. The centripetal force acts at right angles to the instantaneous velocity

DP ready Theory of knowledge

Fictitious forces

You will sometimes see in older books reference to a "centrifugal force". The idea of centrifugal force is that it acts outwards. At first sight this appears quite reasonable: when you are in a car going around a circle then you feel flung outwards from the centre of the circle. However, this feeling is misleading. What you are actually feeling is an effect of Newton 1—you ought to be going in a straight line and the car is forcing you into the circular path. The car exerts a force on you and you experience the *reaction* to this force. Newton 3 tells you that this is opposite to the action force and so the direction of the reaction is away from the centre.

Looking at the situation from above the car shows you what is happening (figure 23).





Chapter summary

Make sure that you have a working knowledge of the following concepts and definitions:

- □ The definitions of distance, displacement, speed, velocity and acceleration and the differences between average and instantaneous values of these quantities.
- □ Distance-time, speed-time, displacement-time and velocity-time graphs can be interpreted to describe motion.
- □ The derivation and use of the *suvat* (kinematic) equations.
- □ There are practical methods for making an estimate of the acceleration due to gravity.
- □ Balanced forces cancel each other out, leaving a body in equilibrium, and forces add to produce one resultant force.
- □ Mass corresponds to the amount of matter in a body, and weight is the gravitational force acting on a body.
- □ Newton's three laws of motion are:
 - 1. every body continues in its state of rest or uniform motion unless external forces act on it
 - 2. the acceleration *a* of an object of mass *m* is related to the force *F* by F = ma or by $F = m \times \frac{\Delta v}{\Delta t}$ where v is the speed of the object and t is the time
 - **3.** every action force has an equal and opposite reaction force.
- Energy is available for use as work when it transfers between energy stores.
- Work done = force \times distance moved in the direction of the force.
- Power is the rate of change of energy with time $\left(-\Delta E \right)$

is the rate of change of energy with time
$$\left(=\frac{1}{\Delta t}\right)$$
.

- Efficiency is $\frac{\text{useful work done}}{\text{total energy transferred}}$ or $\frac{\text{useful power output}}{\text{total power input}}$.
- Energy is transferred from an energy store through an energy pathway to an energy sink.
- Energy is conserved when all final forms, including mass, are taken into account.
- The kinetic energy E_k of an object of mass *m* moving at speed *v* is $\frac{1}{2}mv^2$.
- The change in gravitational potential energy ΔE_{p} of an object of mass *m* is +*mgh* when the object is raised through a distance *h* in a gravitational field of strength (acceleration due to gravity) *g*.
- Momentum = mv and impulse = $F \times \Delta t$, and momentum is conserved.
- In inelastic, elastic and superelastic collisions, the system respectively loses, retains and gains energy.
- For an object to move in a circle, a centripetal force must act on the object towards the centre of the circle, and centripetal force is increased if the speed or mass of the object is increased or if the radius of the circle is decreased.

Additional questions

- 1. A motorboat sails with a velocity of 8.0 m s⁻¹ due north. The wind adds a velocity of 6.0 m s⁻¹ due east. Calculate the overall velocity of the boat in the water. Sketch a diagram to show the direction.
- 2. Two displacements have magnitudes of 18 m and 6 m. Calculate the greatest and least distances of travel that these displacements added together can represent.
- **3.** A force of 9 N and a force of 12 N act on an object. The angle between the forces is 90°.
 - a) Determine, using a scale drawing, the resultant of the two forces.
 - **b)** State how it is possible for the two forces to give a resultant of (i) 3 N and (ii) 21 N.

- **4.** A lorry moves from rest with a constant acceleration of 0.35 m s⁻². The mass of the lorry is 7000 kg.
 - a) Calculate the time taken for the lorry to reach a speed of 17 m s^{-1} .
 - **b)** Calculate the distance travelled by the lorry in reaching the speed of 17 m s^{-1} .
 - c) Calculate the initial force required to accelerate the lorry.
 - **d)** In practice, air resistance acts on the lorry, and the magnitude of the resistive force increases with speed. Suggest what this implies for the speed of the lorry.
- 5. A runner completes a marathon, a distance of 42.2 km, in a time of 3 hours and 30 minutes.
 - a) Calculate, in s, the time taken for the runner to complete the marathon.
 - **b)** Determine the average speed of the runner.
- **6.** A car is initially moving at 32 m s⁻¹. When the brakes are applied its acceleration is -4.6 m s⁻².
 - a) i) Calculate the time taken for the car to stop.
 - **ii)** State the assumption you made in (a)(i).
 - b) The mass of the car is 800 kg. Calculate the resultant force acting on the car.
- **7.** Explain the difference between:
 - a) energy and power
 - **b)** momentum and impulse
 - c) mass and weight.



The graph shows the variation of velocity with time for a ball that bounces vertically after release from rest above the ground.

- a) Explain why the gradient of line OA is equal to the gradient of line BC.
- **b)** Outline why the value of *v* at B is negative.
- c) Explain why the velocity at B is less than the speed at A.
- d) The ball has a mass of 0.25 kg and is released from 1.5 m above the ground. After the first rebound the ball reaches a height of 1.2 m above the ground. Determine:

i) the speed of the ball immediately before the first impact

- ii) the speed of the ball immediately after the first impact.
- **9.** A motor vehicle of mass 950 kg is claimed to travel a distance of 25 m when it stops on a horizontal road from an initial speed of 18 m s^{-1} .
 - a) Determine the average deceleration of the vehicle.
 - b) Calculate the average frictional force that acts on the vehicle.
- **10.** In a test of a motor vehicle's safety, the vehicle is stopped in a distance of 5.5 m from a speed of 28 m s⁻¹. A test dummy of mass 65 kg is wearing a seat belt that allows the dummy to move 0.50 m relative to the vehicle.
 - a) Determine the deceleration of the dummy.
 - **b)** Calculate the resultant force that acts on the dummy.

- 11. An aircraft has a total mass of 3.2×10^5 kg. It is powered by engines that have a total maximum thrust of 1.1 MN.
 - a) Calculate the initial maximum acceleration of the aircraft, ignoring frictional forces.

The aircraft starts its take-off from rest and has a take-off speed of 95 m s⁻¹.

- **b)** Calculate the time to reach take-off speed, ignoring frictional forces.
- **c)** In practice, the frictional forces reduce the acceleration of the aircraft to 2.5 m s⁻². Calculate the mean total frictional force that acts on the aircraft during take-off.
- d) Calculate the minimum length of runway required.
- e) The aircraft travels at a constant velocity and at a constant height after take-off. Explain, with reference to horizontal and vertical forces, how this is achieved.
- 12. A trolley of mass 60 kg is pushed 25 m at a constant speed up a ramp by a force of 55 N acting in the same direction as the direction of motion. The ramp is 2.0 m high. Calculate:
 - a) the work done pushing the trolley up the slope
 - **b)** the change in gravitational potential energy of the trolley
 - c) the energy wasted in friction.
- **13.** A truck in a fairground ride of total mass 1400 kg moves at an initial speed of 2.0 m s⁻¹ before descending through a vertical distance of 50 m to reach a final speed of 28 m s⁻¹. The truck travels on a track of length 70 m in this motion.
 - **a)** Calculate the loss of gravitational potential energy of the truck.
 - **b)** Determine the change in kinetic energy during the motion.
 - c) Determine the average frictional force on the truck during its descent.
- 14. A motor vehicle travels along a horizontal road. When the car travels at a constant velocity of 10 m s^{-1} , its effective power output is 1.8×10^4 W. A resistive force acts on the vehicle. This resistive force consists of two components. One is a constant frictional force and is of magnitude 250 N. The other is the air resistance force and is proportional to the car's speed.
 - a) Determine the total resistive force acting on the vehicle when travelling at a speed of 10 m s⁻¹.
 - b) i) Calculate the force of air resistance when the vehicle is travelling at 10 m s⁻¹.
 ii) Calculate the force of air resistance when the vehicle is travelling at 5.0 m s⁻¹.
 - c) Calculate the effective output power of the vehicle when it is moving at a constant speed of 5.0 m s^{-1} .
- **15.** Estimate the vertical distance through which a 2.0 kg mass would need to fall to lose the same energy as a 35 W lamp will radiate in 90 s.